AN APPROACH TO INTEGRATE CONSTRUCTIVIST LEARNING
THEORY INTO THE TEACHINGS OF MATHEMATICS

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Mathematics is one of the most important and challenging subjects that students have to learn in schools. In addition to this innate difficulty, the needs and expectations of students from schools, in general, and from mathematics, in particular, are changing as society changes. Therefore, school mathematics should be adapted to meet the changing needs of students and society (NCTM, 1989, 1991, 1995; NRC, 1989, 1990 a, b, c, 1991). The Standards (1989) gives guidelines that can be used to make changes in curriculum, instruction, and assessment for different grade levels. One recommendation in this direction was shifting emphasis from a curriculum dominated by memorization of isolated facts and procedures and by proficiency with paper-and-pencil skills to one that emphasizes conceptual understanding, multiple representations and connections, mathematical modeling, and mathematical problem solving (NCTM 1989: 125). Constructivist theories that emphasize active learning might be useful in this shifting process (NCTM, 1989: 127). It is suggested that a learning environment should give opportunity for students to investigate, to make sense of, and to construct meanings from new situations; to make and provide arguments for conjecture; and to use a flexible set of strategies to solve problems both from within and outside mathematics. A classroom environment should be created in which teachers and students are natural partners in developing mathematical ideas and solving mathematical problems. There is a close parallel between what was suggested in The Standards and the assumptions of constructivist learning theories. Simply, constructivist learning theory assumes that the learning environment will promote and encourage the development of each individual with different alternatives to powerful mathematical constructions for posing, exploring,
constructing, solving and justifying mathematical problems and concepts (Confrey, 1990; Brooks and Brooks, 1993).

**Constructivist Approaches**

Constructivism is a philosophical view applied both to how people learn (pedagogy), and to the nature of knowledge (epistemology). It is a popular perspective not only in mathematics education but also in developmental psychology, theories of the family, human sexuality, psychology of gender, and even computer technology (Noddings, 1990: 7). Modern constructivism has its origins in the thinking and writings of Vico in the eighteenth century and owes much of its current conception to the works of Piaget and Bruner in the twentieth century (Jaworsky, 1994). Since constructivist perspective is not settled view in both epistemological and pedagogical terms (Laroche and Bednarz, 1998), there are many different interpretations of constructivism. There are two extremes in the discussion of constructivism: at one end, as some think, constructivist epistemology is the only paradigm that survives, at the other end, there are almost as many varieties of constructivism as there are researchers (Ernest, 1995). Radical, trivial, physical, pragmatic, social constructivism, and social constructionism are some examples of various types of constructivism. Although there are many types of constructivism, this paper focused on the aspects of constructivism that may lead to new models of classroom instructional practices.

Based on Piaget’s work, von Glasersfeld (1990: 22-23) offers the following basic principles of radical constructivism:

1. Knowledge is not passively received either through the senses or by way of communication. The cognizing subject actively builds up knowledge.
2. The function of cognition is adaptive, in the biological sense of the term, tending towards fit or viability; cognition serves the subject’s organization of the experimental world, not the discovery of an ontological reality.

As Jaworsky (1994) pointed out, cognizing subjects in the first principle refers to students in classrooms, the teachers who teach them, the researchers who study them, and readers of this article. Active receiver of knowledge by senses or communication is an important part of the first principle. This contradicts the behaviorist model of learning. According to behavioral theory, learning is conceived as a process of changing or conditioning observable behavior as a result of selective reinforcement of an individual’s response to stimuli that occur in the environment. The mind is seen as an empty vessel, a *tabula rasa* to be filled, or as a mirror reflecting
reality. Behaviorism centers on students’ efforts to accumulate knowledge of the natural world and on teachers’ efforts to transmit it. Therefore, it relies on a transmission, instructionist approach that is largely passive, teacher-directed and controlled (Murphy, 1998). The first principle also suggests that lecturing is likely to be less effective than more active approaches such as cooperative group learning. As a result, learners are likely to make weak constructions. Different students will exhibit different kinds of understanding for the same mathematical task. Although two students may appear to exhibit the same understanding, this may not be the case (Pirie & Kieren, 1992). Therefore, learners are responsible for presenting their ideas about the subject being discussed, defending and justifying their reasoning and communicating their ideas to their classmates. Being adaptive based on the subject’s experiences is an important aspect of the second principle. The concept of adaptation stems from biology. Piaget took the concept of adaptation out of biological context and used it to explain how people learn. His theory paid considerable attention to the construct of adaptation and assimilation. Von Glasersfeld (1995: 374) pointed out that there are different kinds of assimilation. For example, if somebody does not have access to tools to hammer in a nail, and takes a stone for that purpose, knowing that the stone is not a hammer, this person is using the stone as a substitute for something that is not available. He is aware of the difference between a hammer and a stone. Assume that another person takes a wooden mallet for a hammer and tries to hammer in a nail. The nail may go into the mallet rather than into the wall. This discovery generates a change in his concept of the nail, and may lead to a further examination, which then results in an accommodation: the person will create a new category, such as the category of things that look like a hammer but can’t be used for nails. Equilibration is a dynamic process of self-regulated behavior balancing two intrinsic polar behaviors, assimilation and accommodation. The mental growth is a dynamic evolution of more complex mental structures. As a person confronted with a new situation or concept, this causes disequilibrium or cognitive conflict in his mental structure. Accommodation is triggered when the new information does not fit established mental structures, necessitating the adaptation of new structures to pre-existing structures. As Greenes (1995: 87) stated, learning occurs when children’s concepts are challenged by more complex situations, different contexts or by conflicting data or information. Overcoming obstacles, therefore, is a central part of conceptual development. Understanding evolves as children try to make sense of new information, of complexity, of conflict. Concepts are enriched.
as children abstract the salient features of new experiences, and incorporate them into existing mental structures, thereby changing those structures.

Taylor and Campbell-Williams (1993: 4) have offered a third principle to the Von Glasersfeld’s two principals to emphasize the importance of the social construction of knowledge. The third principle "derives from the sociology of knowledge, and acknowledge that reality is constructed intersubjectively; that is, it is socially negotiated between significant others who are able to share meanings and social perspectives of a common." The third principle emphasizes the importance of sociocultural settings in influencing cognition, cooperation, and negotiation with others in offering alternative perspectives and challenging constructions. This last principal is one of the main differences of social constructivism from the radical constructivism. The first two principles are bases for radical constructivism; the third one is the basis for social constructivism. The works of Vygotsky contribute toward the development of social constructivist learning theory (Ernest, 1991). Vygotsky (1981: 63) explained the social development of a child in terms of social and psychological aspects. First, it appears between people as an interspsychological category and then within the child as an intrapsychological category. He did not consider learning as development process but a necessary and universal aspect of the process of developing culturally organized specifically human, psychological functions.

Vygotsky (1978) introduced the concept of the Zone of Proximal Development (ZPD) to provide some measure of a learner’s development related to instruction offered. What the child knows now can be determined by unassisted problem solving, and what functions are in the process of maturing can be determined through assistance with problem solving activity (Greenes, 1995). According to Vygotsky ZPD is "the distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers (1978: 86). Brown and French explained the ZPD as follows:

"... a typical testing session consists of the initial presentation of a test item exactly as it would occur in an American IQ test with the child being asked to solve the problem independently. If the child fails to reach the correct solution, the adult progressively adds clues for solution and assesses how much additional information the child needs in order to solve the problem. The child’s initial performance, when asked to solve the test item
independently, provides information comparable to that gained with standardized American IQ testing procedures. The degree of aid needed before a child reach solution is taken as an indication of the width of his potential zone... the level of competence he can reach with aid. In addition we gain information of the child’s ability to profit from adult assistance, his speed of learning” (1979: 249-58).

This was a new approach to the problem of child’s development that asserted that learning should be matched in some manner with the child’s level of development (Palincsar, 1998). This approach implies that, with appropriate instruction, there may be potential for a child to reach higher conceptual levels than he would be able to achieve naturally (Jaworsky, 1994). ZPD also implies that teaching should not wait until all mental functions required were fully developed (Greene, 1995).

Implications of constructivist approaches to teachings of mathematics

Mathematics is not something that exists independent of human activity, either individually or collectively. Instead, "objective mathematical knowledge" can be viewed as the product of taken-as-shared or human activity, where activity includes both conceptual acts (i.e., thinking) and observable sensorimotor acts. Wood, Cobb, and Yackel (1995) believe that mathematics is the product of an active construction. Each individual is brought into the culture of mathematics and constructs his mathematics based on his previous experiences and through communication with other individuals. Jonassen, Myers, and McKillops (1996) consider constructivism as a process of knowledge construction based on personal experience. They warn, "the teacher can not map his or representation onto the learner, because they don’t share an isomorphic set of experiences and interpretations" (Jonassen et al., 1996: 95). It seems that social constructivism opens up the possibility to look at the interaction between the individual construction and the culture in a positive manner (Richards, 1995).

Although constructivism is not a theory of teaching, it can provide a set of principles that can serve as a guide in the design of learning environments. If mathematics teachers are aware of key elements of constructivism, then, they will be able to adjust their instruction to meet the principles of constructivist learning theory. They can create a constructivist learning environment in which their students value mathematics, become confident in their ability to do mathematics, become mathematical problem
solvers, learn to communicate mathematically, and reason mathematically. Constructivist learning environment is a place where learners may work together and support each other as they use a variety of tools and information resources in their guided effort of learning goals and problem-solving activities (Wilson, 1996). Wilson (1996) explained some of the key elements of constructivist learning environment. A constructivist learning environment should be facilitative of individual knowledge construction; should value multiple perspectives; should be realistic yet relevant; should value individual values; should respect individual learning processes; will use multiple modes of representation; and should support self-awareness of the knowledge construction process.

Mathematics teachers are different in their views of the world and/or the methods of teaching mathematics. Constructivist learning environments can be more easily provided by teachers of certain characteristics that are compatible with this kind of learning setting. Brooks and Brooks (1993), and Yager (1991) suggest some characteristics of constructivist mathematics teachers. They value student autonomy and initiative, use multiple materials and encourages students to use multiple sources, let students responses shape the class, seek these responses even before planning his/lecture, value and support multi-way communication among students, encourage collaborative learning, encourage student questions, particularly open-ended questions, ponder on student responses, support class discussions, provide time for students to construct relationships and create metaphors, make connections between the lecture and real-life problems, encourage out-of-class learning.

Although Yager (1991) also offered some strategies for implementing a constructivist lesson for a science class, these strategies could be used in mathematics classes as well (see table 1). As it can be seen, the role of the teacher is problem or task presenter and facilitator of information-rich environment where students think, explore, discuss, and construct meaning. Teachers present definitions or some concepts related to the subject being taught to have common language for communication between students and teacher, and among students. Then, teachers pose questions or problem situations, and give time to think individually or as a small group. If it is a small group assignment, then the teacher might present a set of guidelines to help the students learn how to cooperate with one another (Davidson, 1990).

Teachers will not be the only source of the right answer/response with the understanding that there is not only one answer to a given problem.
The role of teacher is to offer resources and direction in the course of discussion. Each response will be valued. Students will be encouraged to ask questions, discuss, justify, and defend their opinions with classmates or with the teacher so that agreement can be reached. If there is a viable response that seems most plausible, elegant, or economic as far as calculations and time spent are concerned at the time of discussion, and then the next phase of the subject matter might be presented.

Needless to say, these strategies are general and need to be adjusted depending on the subject matter. For example, someone needs to learn all the

<table>
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<th>Invitation</th>
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<tr>
<td>• Observe surroundings for points to question</td>
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<td>• Ask questions</td>
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<td>• Consider possible responses to questions</td>
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<td>• Identify situations where students' perceptions vary</td>
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<tr>
<th>Exploration</th>
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<td>• Engage in focused play</td>
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<td>• Brainstorm possible alternatives</td>
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<td>• Look for information</td>
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<td>• Design a model</td>
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<td>• Collect and organize data</td>
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<td>• Employ problem-solving strategies</td>
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<td>• Select appropriate resources</td>
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<td>• Discuss solutions with others</td>
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<td>• Engage in debate</td>
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<td>• Analyze the data</td>
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<th>Proposing explanations and solutions</th>
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<td>• Communicate information and ideas</td>
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<td>• Construct and explain a model</td>
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<td>• Review and critique solutions</td>
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<td>• Utilize peer evaluation</td>
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<td>• Assemble multiple answers/solutions</td>
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<td>• Determine appropriate closure</td>
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<td>• Integrate a solution with existing knowledge and experiences</td>
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<th>Taking action</th>
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<tr>
<td>• Make decisions</td>
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<td>• Apply knowledge and skills</td>
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<td>• Transfer knowledge and skills</td>
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<td>• Share information and ideas</td>
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<td>• Ask new questions</td>
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<tr>
<td>• Develop products and promote ideas</td>
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<tr>
<td>• Use models and ideas to illicit discussions/acceptance by others</td>
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Table 1. Summary of Strategies of a Constructivist Lesson
basic operations such as addition, subtraction, multiplication, and division before he tries to find an answer that requires calculations. Knowing these basic operations does not warrant that this person will be able to come up with viable response for the given problem. Students need to be exposed to different problem settings to be able to see when and how these basics can be used. Teachers are the key elements to achieve that goal. Polya (1945) saw teaching not as a science but as an art. He suggested that "you have to present to your class a proof which you know thoroughly having presented it already so many times in former years in the same course. You really can not be excited about this proof-but, please, do not show that to your class; if you appear bored, the whole class will be bored. Pretend to be excited about the proof when you start it; pretend to have bright ideas when you proceed, pretend to be surprised and elated when the proof ends. You should do a little acting for the sake of your students who may learn, occasionally, more from your attitudes than the subject matter presented" (1945: 3).

Asiala, Brown, Devries, Dubinsky, Mathews, and Thomas (1996) classified mental constructions for learning mathematics at three levels: action, process and object. Understanding mathematics begins with manipulating previously constructed mental or physical objects to form actions; actions are then interiorized to form processes that are encapsulated to form objects. These classifications may help the teacher make adjustments on the constructivist learning strategies. They clarified these levels with an example: If a student did not construct function concept, he will be unable to do very much with this function concept except to evaluate it at specific points and to manipulate the formula. Composition, inverses, domains of functions, sets of functions, and the idea that a solution of a differential equation is a function are sources of difficulty for many students because they are unable to go beyond an action concept of function and these notions require process and/or object conceptions. Process conception allows students to think of a function as receiving one or more inputs, or values of independent values, performing one or more operations on the inputs and returning the results as outputs, or values of dependent variables. When a student reflects on operations applied to a particular process, he becomes aware of the process as a totality, realizes that transformations can act on it, and is able to actually construct such transformations. Then he is thinking of this process as an object. At the object level, manipulations of functions such as adding, multiplying, or forming sets of functions are reasonable and not hard to understand. Asiala et al. (1996: 14) also offered three components to get students to reach to the object level: activities, classroom discussions and
exercises (the A. C. E. cycle). Students working in a group lab are exposed
to long activities to gain experience with the mathematical issues that are
later developed in the classroom discussion phase. During classroom
discussions, the instructor provides definitions, explanations and overview to
tie together what the students have been thinking about, and students are
given exercises as homework. The purpose of homework and lab
assignments was to reinforce the ideas they have constructed, to use the
mathematics they have learned, and to begin thinking about situations that
will be studied later.

It is worthy to present Polya’s (1945) four-step problem solving
strategy as facilitator for students to achieve moving from one level (action/
process) to another level (process/object). This four-step approach can be
used not only for solving a given problem, but it can also be used by the
teacher to help students solve the problem and develop the students’ ability
of solving future problem:

1. Understanding the problem: It is the first and most necessary
step to start to answer any question. If a person does not understand, it does
not make sense to expect them to be part of classroom discussion. Students
should have a desire to understand and to try solving the given problem. It is
important to establish a classroom environment in which language being
used is shared and understood by the students. What is the unknown?, what
are the data?, and what is the condition? are common questions that
students should ask of themselves.

2. Devising a plan: Looking at the unknown and similar problems,
using all information and conditions may lead to finding a technique or
method to solve the problem.

3. Carrying out the plan: Students should know basic notations and
rules as part of communication before they can answer the question at hand.
Related concepts, theorems, and rules give a general idea of the subject or
problem being discussed. Students should carry out each step carefully
before moving to next step. Common questions that teachers should ask are,
Can you see that this step is correct? Can you also prove that this step is
correct?

4. Looking back: Teachers might ask the following questions to
examine the result. Can you check the result? Can you justify your
argument? Can you derive the result differently? Can you use the result or
the method for some other problem?
Conclusion
Teaching-learning process is a dynamic bi-directional interaction between teachers and students. Constructivist approaches can be implemented in a classroom setting to facilitate this process. If teachers as facilitators are aware of different aspects of these approaches, they may make adjustments on their instruction and get their students to the point at which students may exhibit their full potential and value what they are learning which we need the most.

REFERENCES


Jonassen, David H., Jamie M. Myers and Ann Margaret McKillop. (1996). “From Constructivist to Constructionism: Learning with


