

# Hurst Analysis of Induction Motor Vibrations from Aging Process

H. Šiljak and S. Şeker

**Abstract**— Different algorithms for Hurst exponent estimation, namely aggregated variance, absolute moment, Higuchi and Peng method, are applied to eight different vibration signals obtained in induction motor aging process. Signals were obtained with accelerometers during an artificial fluting, thermal and chemical aging process. Applicability of Hurst exponent analysis for motor age detection is discussed based on estimation results. Drop of the exponent value for degraded states with respect to the original state is detected, while no monotonic relationship between subsequent states is found. The anti-persistent nature of vibrations is confirmed.

**Keywords**— Hurst exponents; Long-term dependence; Motor vibration; Aging process.

## I. INTRODUCTION

Hurst exponents (Hurst, 1951) as indicators of long-term dependence have usually been used in case of slow-varying natural processes (Rehman, 2007), traffic (Dong et al., 2010) or finance (Qian and Rasheed, 2004). Recently, in pursuit of new features of vibration signals, Hurst analysis has been conducted on machinery vibrations as well (Lin et al., 2012). While in case of slow-varying processes, Hurst analysis is mainly conducted to determine their nature – whether they are long-term dependent (persistent) or not (anti-persistent), in case of vibrations, the main goal is to get a numerical value and compare it with the same parameter in another condition.

Motivation behind this research is the same – checking whether the Hurst exponent may serve as a basis of simple classification algorithm for detecting motor age on one hand, and whether the vibrations show anti-persistent or persistent nature on the other hand. Another important feature of this research is use of different Hurst exponent estimators and choice of the most suitable one for this particular application.

In the second section, basic concepts are covered, namely experimental setup for motor aging processes, the definition of Hurst exponent and ways of estimating it. Third section presents results of different algorithms for Hurst analysis applied to aging process vibration signals, together with a

brief discussion of results. Finally, conclusions are drawn in the last section. Together with conclusions, a suggestion for future research is given, in order to potentially generalize these results.

## II. BASIC CONCEPTS

For a complete grasp of material covered in this paper, certain introduction to motor aging experimental techniques and the theory of Hurst exponents is needed. Therefore, this section gives the basic information on those topics.

### A. Motor aging process experiment setup

An accelerated aging test that had been performed according to IEEE-Std 117 (1974) test procedures gave results in form of eight vibration time series (Erbay, 1999). It was conducted as a combination of fluting, thermal and chemical aging. Namely, fluting aging simulates the electrical discharge from the shaft to the bearing by externally applying shaft current of 27 A at 30 V AC for 30 minutes, while the motor runs with no load.

This procedure was conducted in seven runs before the motor failed in the end of seventh one. This way, eight data sets for motor vibration were collected (first one being the initial case). Sampling frequency is 12 kHz, measurement time 10 s and an anti-aliasing filter at 4 kHz has been applied. Figure 1 shows the first vibration signal record from the experiment.

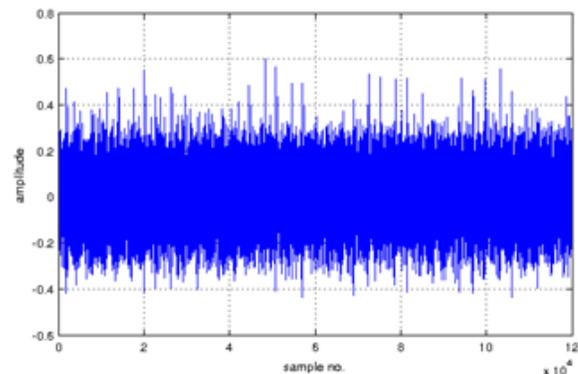


Figure 1. First (out of eight) vibration signal collected in the experiment

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**B. The Hurst Exponent**

As defined in (Hurst, 1951), H (Hurst exponent, self-similarity coefficient) is described by the asymptotic relation

$$E \left[ R \frac{(n)}{S(n)} \right] \sim n^H, n \rightarrow \infty \tag{1}$$

Here,  $R_{(n)}/S_{(n)}$  is the rescaled range ( $R_{(n)}$  is difference between maximum and minimum term in the time series and  $S(n)$  is the standard deviation).

For  $0.5 < H < 1$  it is said that process has long-range dependence, for  $H=0.5$  it is uncorrelated, while for  $0 < H < 0.5$  the process has short-range dependence (Beran, 1994). In simple terms, this means that high values of H correspond to processes which highly depend on previous values and have memory (persistent) while low values express the opposite, being anti-trended (anti-persistent).

Hurst originally used this limit as a method of Hurst exponent calculation in his work on yearly Nile high water (Hurst, 1951).

**C. Methods for Hurst exponent estimation**

There are various methods proposed for Hurst exponent estimation besides the one originally proposed by Hurst (shown in previous subsection). In this paper, four methods are used, namely Aggregated variance, Absolute moment, Higuchi and Peng methods. Description of those methods is adapted from (Taqqu et al., 1995), while the implementations used in this paper are MATLAB functions from (Chen, 2008). All examples of method plots have been made using the vibration data in normal motor state (the one shown in Fig. 1) applying the corresponding MATLAB functions.

First one, Aggregated variance method is using aggregated series

$$X^{(m)}(k) = \frac{1}{m} \sum X(i), k=1,2,\dots \tag{2}$$

where m is the block size and k is the block label. Taking sample variance of  $X^{(m)}(k), k=1,2,\dots$  within blocks gives an estimate for H since  $\text{Var}X^{(m)} \sim m^{2H-2}$  as  $m \rightarrow \infty$ . Plotting the logarithm of variance vs logarithm of block size should give a line with slope  $2H-2$ . Example of such procedure is shown in Fig. 2.

Next method is a similar one. For Absolute moment approach, one finds the sum of the absolute values of aggregated series

$$\frac{1}{N/m} \sum |X^{(m)}(k)| \tag{3}$$

which plotted in log scale vs logarithm of block size should give a line with slope . This is shown in Fig. 3 in the same manner as before.

Higuchi method requires calculation of

$$L(m) = \frac{N-1}{m^3} \sum \left[ \frac{N-i}{m} \right]^2 \sum | \sum X_j - \sum X_j | \tag{4}$$

where N is the time series length and m block size. Since it is  $L(m) \sim m^{H-2}$  for  $m \rightarrow \infty$ , slope of the logarithm plot is  $H-2$ . This is shown in Fig. 4.

Finally, Peng method does a procedure equivalent to calculating sample variance of the time series. Since  $\text{Var}X \sim m^{2H}$  for  $m \rightarrow \infty$ , logarithm plot gives a line with a slope of  $2H$ . It is shown in Fig. 5.

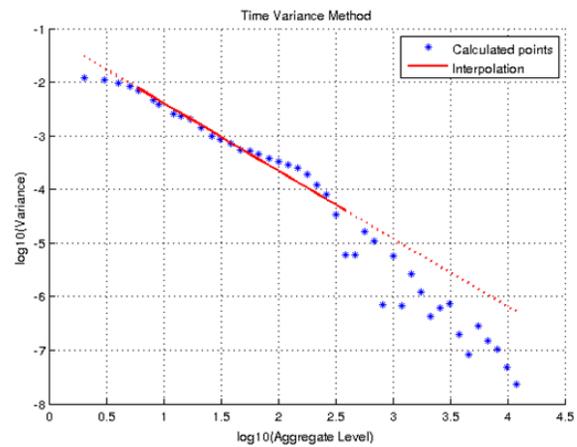


Figure 2. Aggregated variance approach applied to signal in Fig. 1

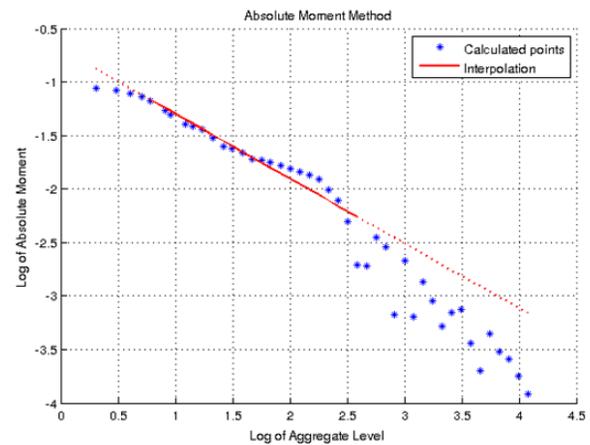


Figure 3. Absolute moment approach applied to signal in Fig. 1

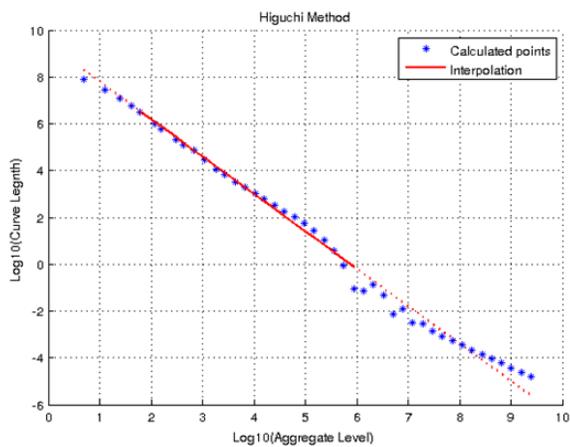


Figure 4. Higuchi approach applied to signal in Fig. 1

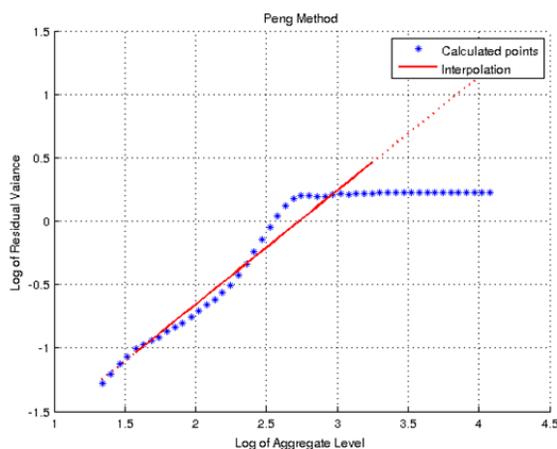


Figure 5. Peng approach applied to signal in Fig. 1

This interval ends when capacitor voltage decreases to zero and output diode turns on.

### III. RESULTS AND DISCUSSION

Figure 5 and Table 1 represent the summary of our results. As expected,  $H < 0.5$  regardless of the algorithms and data sets. However, no monotonicity is found – absence of any trend is clearly shown in Figure 5 with a sine-like curve. The numerical values in each of the algorithms do not match. Still, that is not influencing our results, since the overall behavior is the same. By inspection (see Figs 2-5) we may also conclude that the Higuchi method can be taken as the most precise in this case.

Expectation of anti-persistent behavior confirmed here is based on the definition of vibration itself – representing a dynamic excitation whose duration is substantially longer than the response time of the system exposed to it (Harris and Piersol, 2010).

TABLE 1. OBTAINED VALUES FOR HURST EXPONENTS IN DIFFERENT AGING CYCLES AND DIFFERENT METHODS

Algorithm/cycle	0	1	2	3	4	5	6	7
Absolute moment	0.3951	0.3089	0.2892	0.2924	0.3546	0.3689	0.3043	0.3010
Aggregated variance	0.3696	0.2825	0.2695	0.2790	0.3355	0.3451	0.2899	0.2814
Higuchi	0.4016	0.3190	0.2993	0.2997	0.3569	0.3739	0.2980	0.3463
Peng	0.4484	0.3505	0.3296	0.3362	0.3971	0.4101	0.3333	0.2952

It is not clear whether the sine-like behavior of Hurst exponent for different aging cycles is characteristic to these processes in general, but it clearly falsifies a potential hypothesis that Hurst exponent rises or falls monotonically throughout the aging process in whole, although the value of Hurst exponent is clearly lower in every aged state than in original (dataset zero) state, as implied by (Ikizoglu et al., 2010).

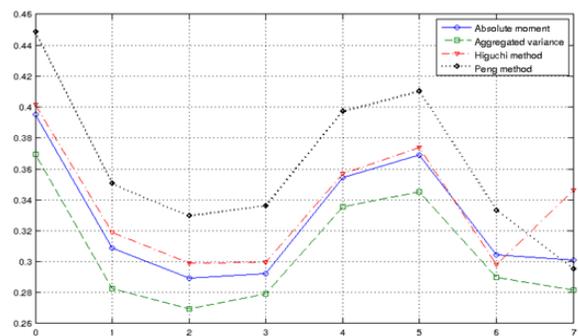


Figure 6. Graphical representation of obtained Hurst exponent values

### IV. CONCLUSION

In this paper, Hurst exponents analysis has been applied to motor vibrations with partial success. While the results have shown to be in accordance with theoretical expectations in the matter of signal nature, potential application of this parameter in predictive maintenance does not seem straightforward. Apparent lack of correlation between the motor age and long-term dependence of its vibrations excludes possibility of a simple deduction of its state by looking for the trend of Hurst exponent change. Nevertheless, if similar sine-like behavior is detected for other aging process vibrations, then it may serve as a base of a more complex classification approach. More research has to be conducted in this sense.

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## REFERENCES

- [1] J.Beran, *Statistics for Long-Memory Processes*. Chapman & Hall, New York, 1994.
- [2] K. Q.Dong, P. J. Shang, and H.Zhang, "The Multi-Dependent Hurst Exponent in Traffic Time Series", *Applied Mechanics and Materials*, vol. 20-23, 2010, pp.346-351.
- [3] A. S. Erbay, *Multi Sensor Fusion for Induction Motor Aging Analysis and Fault Diagnosis*, Ph.D. dissertation, The University of Tennessee, Knoxville, 1999.
- [4] H.E.Hurst, Long-term storage of reservoirs: an experimental study. *Transactions of the American Society of Civil Engineers*, vol.116, 1951, pp.770-799.
- [5] Hurst parameter estimate (by Chu Chen) (2008), Last Accessed on Feb 28, 2013. Hyperlink: <http://www.mathworks.com/matlabcentral/fileexchange/19148-hurst-parameter-estimate>,
- [6] S.Ikizoglu, R.Caglar, and S.Seker, "Hurst Parameter and Fractal Dimension Concept for Fault Detection in Electric Motors", *Int. Rev. of Electrical Eng. Part A*, Vol.5, No.3, 2010, pp. 980-984.
- [7] J.Lin, Q. Chen, X. Tian, and F.Gu, "Fault Diagnosis of Rolling Bearings using Multifractal Detrended Fluctuation Analysis and Mahalanobis Distance Criterion", *Proc. of the 18th International Conference on Automation and Computing (ICAC)*, 2012, Loughborough, UK.
- [8] B.Qian, and K.Rasheed, "Hurst Exponent and financial market predictability", *IASTED conference on Financial Engineering and Applications, FEA 2004*, pp.203-209
- [9] S.Rehman, "Study of Saudi Arabian climatic conditions using Hurst exponent and climatic predictability index", *Chaos, Solitons & Fractals* vol.39, no.2, 2009, pp. 499-509.
- [10] M. S. Taqqu, V. Teverovsky, and W.Willinger, "Estimators for Long-Range Dependence: An Empirical Study", *Fractals*, vol.3, 1995, pp. 785-798.

## BIOGRAPHY



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