Entropy Generation Minimization in Thermoelectric Heat Pump Systems with Multi-channel Heat Exchangers

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Abstract

This work aims to propose routes for thermoelectric heat pump design based on Entropy Generation Minimization (EGM). The system considered is composed of a thermoelectric module sandwiched between two parallel multi-channels heat exchangers. Co-optimization of the heat exchangers and the thermoelectric module designs are lead in order to increase the system performance, depending on realistic manufacturing constraints. The optimized variables considered in this study are: the thermoelectric leg number, their length and section; and the number of channels and their length and diameter in both hot and cold heat exchangers. The following dissipative contributions are identified: thermal conduction and Joule effect in the thermoelectric module, and heat transfer and viscous dissipation in both heat exchangers. On the one hand, the thermoelectric module design has to respect optimal design ratios derived analytically to meet a given thermal power demand with maximum COP. On the other hand, dissipative contributions competition in the heat exchangers results in two distinct optimal designs, depending on the thermal power density. Taking into account manufacturing constraints, realistic system design is derived and discussed. Although the entropy generation in the heat exchangers is low compared to that in the thermoelectric module (dominated by Joule effect), the heat exchanger design highly impacts the global system performance.

Keywords: Second law analysis; entropy generation minimization (EGM); energy conversion; heat exchanger; viscous dissipation.

1. Introduction

Since thermoelectric phenomena were discovered at the end of 19th century, many thermoelectric applications have been considered to convert electric power to thermal power, and inversely [16]. These thermoelectric devices are generally used when the target applications impose minimum space, noise, weight or an environment with substantial mechanical constraints. However, thermoelectric heat pumps (THPs) still have low performances compared to classical vapor compression heat pump, since the system design and its integration are crucial.

Many studies have been lead on the thermoelectric module itself to characterize its intrinsic performance. Thermodynamic analysis in the thermoelectric material is presented in [4]. Using numerical and analytical models, several authors [10,11,13] showed that equivalent performance can be reached with various thermoelectric module designs. In particular, the number of legs, their section and their length are closely related so as to meet a given thermal power demand with maximum COP, depending on the thermoelectric operating temperatures.

Moreover, the system performance highly depends on the thermal coupling between the thermoelectric module and the heat source/sink via the heat exchangers. A detailed analysis of the internal and external irreversibilities is proposed by Kaushik [9] Energy and exergy study of a thermoelectric ventilator (including heat exchangers) for building applications is also given in [6] for variable operating conditions. Based on entropy generation analysis, heat exchanger design optimization is discussed in Wang [18] but the pressure drop dissipation in the heat exchangers is neglected.

On the other hand, a detailed thermodynamic analysis of heat exchangers is proposed by Feidt [3]. Entropy generation minimization analysis shows that the optimal design of heat exchangers is a compromise between both dissipative contributions, i.e. thermal transfer and viscous dissipation. However, viscous dissipation is regularly neglected in the previous studies on thermoelectric systems and few studies [2] are carried out on the design of the whole system, including all the dissipation contributions related to the thermoelectric module and the two heat exchangers. A similar approach is proposed in [14] to design thermoelectric generators.

More recently THPs have been used for heating and cooling devices for buildings [5], since thermal power needs and the temperature difference are decreasing with the advent of low-temperature heating devices. Particular attention is paid to this type of application, for which the compactness of the THP is not the key constraint. Lower thermal power density can thus be achieved, leading to reconsider the classical THP design originally aimed for high power density.

This study aims to discuss the optimal design of THPs dedicated for building applications in an attempt to increase the system’s performance. The THP considered is composed of a thermoelectric module sandwiched between two heat exchangers, dedicated to water/water building heat pump applications. The thermoelectric module is classically composed of several legs, made of bulk semiconductor
The authors propose to co-optimize the thermoelectric module and the heat exchangers designs thanks to entropy generation minimization. The optimization is performed by taking into account the following dissipative contributions: Joule effect and thermal conduction in the thermoelectric module; thermal transfer and viscous dissipation in both heat exchangers. Their comparison and analysis help to propose realistic THP design depending on the thermal power demand, the source/sink temperatures and the thermoelectric material properties, with respect to manufacturing constraints.

2. Model

The thermoelectric heat pump uses Seebeck effect in semi-conductors material to transfer thermal energy from a cold source to a hot sink via the hot and cold heat exchangers. The system considered is composed of a planar thermoelectric module sandwiched between two multi-channels water heat exchangers, as presented in Figure 1.

![Figure 1. Schematic representation of the system.](image)

The engineering optimization problem can be formulated as follow: What is the best system design to reach a thermal demand with maximum performances? This formulation assumes that the thermoelectric material and fluid properties are known. The operating conditions, such as the inlet/outlet temperatures in both heat exchangers and the thermal demand, have to be defined with regard to the application aimed. The design variables considered in this study are:

- In the thermoelectric module: the leg length $L_l$, its section $A_l$ and their number $N_l$.
- In both heat exchangers (assuming identical design): The number of channels $N$, their diameter $D$ and length $L$.

The model was simplified according to the following main assumptions that help to interpret the main evolution of the system regarding to the internal dissipative phenomena:

- Uniform fluid temperature is assumed in both heat exchangers, taken equal to the outlet temperature. Under this assumption, the hot and cold temperatures are assumed uniform at the thermoelectric junctions. This assumption helps to set homogeneous operating conditions to the thermoelectric module (thermal constrictions resistances neglected). For further precision, a 3D space meshing would be needed [1].
- By the way, the hot and cold junction temperature difference is maximum that implies unfavorable operating conditions of the thermoelectric module [2], leading to over-estimate the entropy generation in the module.
- Heat exchanger fluid distributor and collector are not considered in this study. The flow is assumed fully-developed from the inlet to the outlet of heat exchanger channels. The validity of this assumption is discussed in section 3.2.

2.1 Thermoelectric module

The thermoelectric description is based on the model originally proposed by Ioffe [8]. Let’s consider a thermoelectric module composed of $N_t$ thermoelectric legs connected in series electrically and in parallel thermally. The legs of section $A_l$ and length $L_l$ are made of bulk semiconductor materials (mainly Bismuth Telluride for working temperatures close to the ambient temperature).

The leg properties (Seebeck coefficient $\alpha$, thermal conductivity $\lambda$ and electrical resistivity $\rho$) are evaluated at a mean temperature $T_m = \frac{T_h^{TE} + T_c^{TE}}{2}$, where $T_h^{TE}$ and $T_c^{TE}$ are the hot and cold junction temperatures, respectively. This assumption remains valid as long as the Joule effect is not excessive [17]. Besides, the temperature difference between $T_h^{TE}$ and $T_c^{TE}$ is usually low when thermoelectric elements operate in THP mode, so $T_m$ is close to the mean side temperatures. The present study is based on the expressions for Bi$_2$Te$_3$ bulk properties depending on the temperature (Table 1), given by Riffat [15].

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal conductivity $\lambda(T)$</td>
<td>$(62.605 - 277.7T + 0.4131T^2).10^{-4}$</td>
</tr>
<tr>
<td>Electrical resistivity $\sigma(T)$</td>
<td>$(5.112 + 163.4T + 0.6279T^2).10^{10}$</td>
</tr>
<tr>
<td>Seebeck coefficient $\alpha(T)$</td>
<td>$(22.224 + 930.6T + 0.9905T^2).10^{-9}$</td>
</tr>
</tbody>
</table>

Assuming that Joule effect is fairly distributed between hot and cold junctions in steady state [8], the released and absorbed heat fluxes at the thermoelectric junctions, respectively $Q_h$ and $Q_c$, are the sum of three contributions: Seebeck effect, Joule effect and thermal conduction.

\[
Q_c = Q_c^2 + Q_c^1 - Q_c^3 = N_l \left( \alpha_m T_c^{TE} - \frac{1}{2} R I^2 - K \Delta T \right) \quad (1)
\]

\[
Q_h = Q_h^2 + Q_h^1 - Q_h^3 = N_l \left( \alpha_m T_h^{TE} + \frac{1}{2} R I^2 - K \Delta T \right) \quad (2)
\]

The electrical resistance $R = L_l/A_l \sigma$ and the thermal conductance $K = A_l/L_l \lambda$ of the legs are determined by integration on a leg of length $L_l$ and section $A_l$. In accordance with the first law of thermodynamics, the electrical power $W_e$ to supply to the thermoelectric module is deduced from $Q_h$ and $Q_c$:
\( W_e = Q_h - Q_c = \Delta V \cdot I = N \varepsilon (\alpha_m \Delta T + R I^2) \)  

(3)

The heating COP is defined as the ratio between the released heat and the electrical power supplied to the TE:

\[
COP = \frac{Q_h}{W_e} = \frac{\alpha_m R I^2 + \frac{1}{\alpha_m} \Delta T}{\Delta T + R I^2}
\]

(4)

Applying the second law of thermodynamics to the thermoelectric module leads to:

\[
\frac{Q_h}{T_h} + \frac{Q_c}{T_c} + S_{\text{gen}}^{\text{TE}} = 0
\]

(5)

With respect to Eqs. (4) and (5), the COP is closely related to the entropy generation in the system by:

\[
COP = \frac{\frac{Q_h}{T_h}}{\frac{Q_c}{T_c}} \frac{T_h}{T_c} - S_{\text{gen}}^{\text{TE}}
\]

(6)

As demonstrated by Goupil [4], the entropy generation in thermoelectric systems is the sum of two contributions: (i) Thermal conduction and (ii) Joule effect, so that:

\[ S_{\text{gen}}^{\text{TE}} = S_{\text{gen}}^{\text{Cond}} + S_{\text{gen}}^{\text{Joule}} > 0 \]

(7)

Both contributions to the entropy generation are given by their respective entropy flux:

\[ S_{\text{gen}}^{\text{Cond}} = Q^{\text{Cond}} \left( \frac{1}{T_c} - \frac{1}{T_h} \right) > 0 \]

\[ S_{\text{gen}}^{\text{Joule}} = Q^{\text{Joule}} \left( \frac{1}{T_c} + \frac{1}{T_h} \right) > 0 \]

(8)

(9)

2.2 Heat Exchangers

Two heat exchangers are needed to transfer heat from the source/sink to the thermoelectric module at the hot and cold sides, respectively. Heat transfer is ensured by water flow in several parallel multi-channels in both heat exchangers, as presented in Figure 1.

The first law of thermodynamics applied to an incompressible fluid leads to:

\[ Q = \frac{\rho \Delta h}{T_h} + \dot{m} \Delta S + S_{\text{gen}}^{\text{hhx}} = 0 \]

(10)

This equation is used for both hot and cold heat exchangers with subscripted notations c/h. The pressure drop across each heat exchanger is evaluated thanks to following classical correlations [7] for fully developed flow in channel:

For laminar regime \((Re < 2300)\):

\[ f = \frac{64}{Re} \]

For turbulent regime \((3000 < Re < 5.10^6)\):

\[ f = \frac{1}{0.790 \ln(Re) - 1.642} \]

Concerning the transitional regime, for value of \(Re\) included in the range \([2300; 3000]\), \(f\) is linearized for numerical convergence purpose. Note that the Reynolds number \(Re = \frac{\rho v L}{\mu}\) is estimated with the channel diameter \(D\) and the flow velocity \(v = \frac{\rho \dot{m}_{ch}}{\pi D^2/4}\), assuming an equidistribution of the total mass flow between the channels, so that \(\dot{m}_{tot} = N \dot{m}_{ch}\). The pressure drops is then evaluated with respect to the friction factor \(f\), as:

\[ \Delta P = \rho \frac{L}{D^2} \frac{v^2}{2} \]

(11)

Thermal transfer through the heat exchangers results in a thermal resistance between the junction temperature and the fluid temperature, mainly caused by convection. Thermal conduction in the heat exchanger bulk is neglected as its contribution to the global thermal resistance is very low compared to convective contribution (for high conductive material). Considering only convective heat transfer in the mini-channels, it can be written:

\[ Q = h A_{\text{ex}} (T_h - T_{out}) = \frac{T_h - T_{out}}{R_{\text{ch}}} \]

(12)

where \(T_h\) is the temperature of each heat exchanger basis, taken equal to the hot and cold junction temperatures for the hot and cold heat exchangers, respectively. The exchange area is \(A_{\text{ex}} = \pi DN_{\text{ch}} L\) and the footprint area of the heat exchanger is \(A_{\text{hx}} = 2DN_{\text{ch}} L\), assuming the \(N\) channels of diameter \(D\) and length \(L\) are spaced regularly with a minimal interstice equal to their diameter \(D\). The heat exchanger width is thus given by \(l = A_{\text{hx}} / L = 2N_{\text{ch}} D\).

The convective heat coefficient is evaluated thanks to the common correlation of literature [7]:

For laminar regime \((Re < 2300)\):\(Nu = 4.36\)

For turbulent regime \((Re > 10^4)\):\(Nu = 0.023Re^{0.6}Pr^{0.4}\)

with \(Nu = h D / \mu\) and \(Pr = \mu C_p / \lambda \approx 7\) for water around ambient temperature. As for the friction factor \(f\), a linear interpolation is used for transitional regime.

Applying the second law of thermodynamics to the heat exchanger system leads to:

\[ \frac{Q}{T_h} + \dot{m} \Delta S + S_{\text{gen}}^{\text{hhx}} = 0 \]

(13)

where the entropy variation of water from the inlet to the outlet is:

\[ \Delta S = C_p \ln \left( \frac{T_{in}}{T_{out}} \right) \]

As detailed in [3], the entropy generation in the heat exchanger is caused by two additional dissipative phenomena: heat transfer and viscous friction. Their respective contributions to the global entropy generation in both heat exchangers are:

\[ S_{\text{gen}}^{\text{th}} = \frac{Q}{T_h} \left( \frac{1}{T_h} - \frac{1}{T_{out}} \right) > 0 \]

\[ S_{\text{gen}}^{\text{lp}} = \dot{m} C_p \ln \left( \frac{A_{\text{ex}}}{\rho C_p T_{in}} + 1 \right) > 0 \]

(14)

(15)

3. Results and Discussion

3.1 Simulation Conditions

Based on the mathematical model presented in the previous section, simulation tools have been developed with the software Engineer Equation Solver (EES) for both heat exchangers and the thermoelectric module.

Simulations were performed considering operating conditions closed to a water/water thermoelectric heat pump coupled to a ground source heat exchanger (as the cold source) and used for low temperature building heating.
(Figure 1). The whole system has to supply a given thermal power $Q_{\text{demand}}$ to meet the building heating demand. For these simulations, the heat exchangers fluid outlet temperature $T_{\text{out}}$ and pressure $P_{\text{out}}$ have been set (for both hot and cold sides). The temperature difference from the inlet to the outlet in the heat exchangers is usually few K for this kind of application and has been set at 5 K for the hot heat exchanger. The resulting mass flow rate is also used in the cold heat exchanger. Consequently, when considering the whole system, the temperatures $T_h^T_E$ and $T_c^T_E$ and the thermal fluxes $Q_h$ and $Q_c$ at the thermoelectric legs and heat exchangers interfaces are not imposed and are determined by the thermal coupling between these components. The operative conditions considered in this study are summarized in Table 2. The water thermophysical properties are assumed constant.

At first, numerical optimization has been performed on the hot heat exchanger itself to discuss the optimal designs depending on its footprint. Secondly, the design of the thermoelectric module is discussed with regards to the thermal resistance of the heat exchangers. Based on these component analyses, an optimization method has been carried out to determine the optimal design of the whole system. The optimization procedure based on the minimization of the entropy generation in the system considered is performed with the Variable Metric Method [12] algorithm available in the EES solver software. The convergence criteria have been set to $10^{-12}$ for the relative residual and $10^{-10}$ for the relative convergence tolerance. Independence of the convergence from initial guess has been checked (not presented here for sake of brevity).

### Table 2: THP operating condition.

<table>
<thead>
<tr>
<th>Operating condition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{\text{demand}}$</td>
<td>3 000 W</td>
</tr>
<tr>
<td>$P_{\text{out}}$</td>
<td>0.2 MPa</td>
</tr>
<tr>
<td>$T_{\text{out}}$</td>
<td>303.15 K</td>
</tr>
<tr>
<td>$T_c$</td>
<td>278.15 K</td>
</tr>
<tr>
<td>$\Delta T_{hx}$</td>
<td>$[T_{hx}^h - T_{hx}^c]$</td>
</tr>
<tr>
<td>$\Delta V$</td>
<td>48 V</td>
</tr>
</tbody>
</table>

#### 3.2 Heat Exchanger Analysis

Heat exchanger analysis is lead on the hot heat exchanger for the operating conditions given in Table 2. The optimization variables considered in this part and their bounds are reported in Table 3.

### Table 3: Heat exchanger optimization variable range.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unit</th>
<th>Min. Value</th>
<th>Max. Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>-</td>
<td>1</td>
<td>$5 \times 10^4$</td>
</tr>
<tr>
<td>$D$</td>
<td>m</td>
<td>$10^{-3}$</td>
<td>$10^{-2}$</td>
</tr>
<tr>
<td>$L$</td>
<td>m</td>
<td>$5 \times 10^{-1}$</td>
<td>$10^1$</td>
</tr>
<tr>
<td>$N_h$</td>
<td>-</td>
<td>2</td>
<td>$1 \times 10^4$</td>
</tr>
<tr>
<td>$L_h$</td>
<td>m</td>
<td>$10^{-3}$</td>
<td>-</td>
</tr>
<tr>
<td>$A_h$</td>
<td>m²</td>
<td>$10^5$</td>
<td>$10^3$</td>
</tr>
</tbody>
</table>

Two competitive dissipative phenomena, associated to heat transfer and to viscous friction, occur in a heat exchanger. The optimal design of a heat exchanger is thus a compromise between heat transfer intensification and viscous friction reduction [3].

Figure 2 plots the total heat exchange entropy generation $S_{\text{gen}}^{hx}$ and both contributions $S_{\text{gen}}^{\text{th}}$ and $S_{\text{gen}}^{\text{v}}$ of the optimized heat exchanger designs, for one hundred values of footprint $A_{hx}$ varying exponentially from 0.05 to 50 m². Note that the thermal power density $dQ = Q/A_{hx}$ is inversely proportional to the heat exchanger footprint, as the thermal power needs is set. As expected, increasing thermal power density implies increasing entropy generation in the heat exchangers. Both entropy generation contributions are also monotonically increasing functions and show a singularity around 3000 W/m² (Aₕ, around 1 m²). Note that entropy generation due to thermal transfer highly dominates entropy generation caused by viscous friction.

The Bejan number ($Be = \frac{S_{\text{gen}}^{\text{th}}}{S_{\text{gen}}^{\text{th}} + S_{\text{gen}}^{\text{v}}}$) varies from 0.85 to 0.91 in configuration 2 and is almost constant close to unity in configuration 1.

The optimization proposed leads to two distinct designs depending on the thermal power density. As this transition is difficult to determine accurately, a “no confidence area” is plotted to highlight this numerical difficulty. To discuss the optimal heat exchanger designs, the resulting optimized variables are reported in Figure 3 and 4. The resulting thermal resistance of the heat exchanger is reported in Figure 2. It can be checked that the fluidic and thermal entry length [7] remains, at least, 10 times lower than the channel length (for any heat exchanger design). The flow is thus fully-developed in the major part (>90%) of the channel, that strengthen the use of fully-developed correlations for the estimation of Nusselt and friction factor in the channel.

For high heat exchanger footprint or low thermal power density (configuration 1), the optimization is driven by viscous dissipation, as thermal transfer is not limiting. The optimization procedure tends to reduce the channel length $L$ to its minimum (and thus increase the heat exchanger width $l$) to minimize viscous dissipation contribution. Based on the model presented above, it can be shown that the heat exchanger width $l$ is inversely proportional to the Reynolds number $Re$ in the channel. By the way, this design results in low Reynolds number that ensures laminar flow in the channels. Then, the optimization procedure tends to decrease the channel diameter $D$, as the Nusselt number $Nu$ remains constant in laminar flow. It results in high number of channels $N$ to reach significant heat exchanger width $l$ (set by the low exchanger length and the high total footprint). Hence, the number of channels $N$ decreases with increasing thermal power density.

For lower heat exchanger footprint or higher thermal power density (configuration 2), the optimization is driven by the thermal dissipation, resulting in high channel length $L$. This optimal design implies low heat exchanger width $l$ and thus high Reynolds number $Re$, i.e. turbulent flow in the channels. Optimal Reynolds number remains around $10^5$, corresponding to the transitional/turbulent regime transition for the Nusselt number $Nu$. This optimal value may result from the linear interpolation of Nusselt against Reynolds for transitional regime. On the other hand, the optimization procedure tends to decrease the number of
channel $N$ to its minimum, in order to limit the impact of viscous dissipation. Finally, this optimal design correspond to the minimum number of channels $N$, the optimal channel diameter $D$ is set by the optimum Reynolds $Re=10^3$, giving high channel diameter. The optimum leg length $L$ is then conditioned by the heat exchanger footprint (or thermal power density).

The change in the optimal design results from the competition of both entropy generation contributions. This singularity is observed for a thermal power density around 3000 W/m² (Reynolds number around 200), in our simulation conditions. This bound is obviously highly dependent of the variables range considered. For low thermal power density (i.e. high heat exchanger footprint), laminar flow configuration (configuration 1) is preferred, as it leads to lower entropy generation in the heat exchangers (compared to configuration 2). However, for high thermal power density (i.e. low heat exchanger footprint), laminar flow in the channels is not efficient enough to ensure sufficient thermal transfer and turbulent flow is thus needed (configuration 2). Removing the design constraints on the design variables (higher ranges) would result in heat exchanger optimal design with high number of parallel channels of small diameter and length (configuration 1 with laminar flow).

### 3.3 Thermoelectric Module Analysis

This part focuses on the thermoelectric module design optimization with the aim to reach the maximum COP for a given hot useful thermal power $Q_h^*$. To keep the junction temperatures as coupling variables with respect the operating conditions defined in Table 2, the thermoelectric module design is analyzed depending on the thermal resistance $R_{th}$ between the thermoelectric junction temperatures and the source/sink temperatures (heat exchanger design not considered here). As the useful thermal power $Q_h^*$ and the source/sink temperatures are assumed to be known, setting a thermal resistance gives the thermoelectric junction temperatures $T_{th}^{TE}$ and $T_{c}^{TE}$ (derived from Eq. (4) for maximum COP). The design variables considered in this part restrict to the thermoelectric module. The variable ranges considered are summarized in Table 3.

The optimization problem can be partly solved with the following analytical considerations. The optimal electrical current that gives the maximum COP (or minimum $S_{gen}^{TE}$) is derived analytically with $\frac{\partial COP}{\partial I} = \frac{\partial S_{gen}^{TE}}{\partial I} = 0$:

$$I^* = \frac{k \Delta T}{\alpha m T_m} (M + 1) \tag{16}$$

With $M = \sqrt{1 + Z T_m}$ and $Z = \frac{\alpha m}{A}$, thermoelectric material constants.

As the operating temperatures and the material properties are supposed to be known, setting the optimal electrical current, Eq. (16) links the optimal electric current $I^*$ to the optimal leg length $L_t^*$ and optimal section $A_t^*$ via the constant ratio $R_{th}^{TE}$, as follows:

$$R_{th}^{TE} = \frac{I^* L_t^*}{\frac{A_t^*}{S_{gen}^{TE}}} = \frac{\lambda \Delta T}{\alpha m T_m} (M + 1) \tag{17}$$

In this condition, the maximum COP is given by:

$$COP^* = \frac{T_m}{\Delta T} \left( \frac{M-1}{M+1} + \frac{1}{2} \right) \tag{18}$$

Note that these values only depend on the thermoelectric material properties and the operating junction temperatures (set by the source temperatures and
the thermal resistance of the heat exchangers). That is to say, the maximum COP is independent of the thermoelectric design and identical performances can be reached while the design complies with the ratio $R_{22}^T$. Furthermore, the thermoelectric design has to cover the heating power demand with optimal electrical current given by:

$$Q_h^* = \frac{N_L R_{22}^T}{T_m} \left[ T_{h}^{TE} (M + 1) + \frac{\Delta T}{2} \left( \frac{M+1}{M-1} - 1 \right) \right]$$  \hspace{1cm} (19)

It follows that the optimal electric current $I^*$ and the optimal legs number $N_L^*$ are linked by the following constant ratios $R_{12}^T$ and $R_{34}^T$:

$$R_{12}^T = I^* N_L^* = \frac{Q_h^* (M+1)}{\sigma_m T_h^{TE} (M+1) + \frac{\Delta T}{2} \left( \frac{M+1}{M-1} - 1 \right)}$$  \hspace{1cm} (20)

$$R_{34}^T = \frac{I^*_2}{N_L^* N_L^*} = \frac{\Delta T}{Q_h^{TE} (M+1)} \left( T_{h}^{TE} (M + 1) + \frac{\Delta T}{2} \left( \frac{M+1}{M-1} - 1 \right) \right)$$  \hspace{1cm} (21)

Any thermoelectric module design that respects the previous ratios $R_{12}^T$, $R_{22}^T$ and $R_{34}^T$ will meet the useful thermal power $Q_h^*$ with maximum COP [13]. Similar expressions including Thomson effect are also given. Analytical developments allow to express the minimum entropy generation contributions in the thermoelectric module (for a given thermal demand at optimal electrical current) as:

$$S_{\text{Cond}} = \frac{\Delta T}{R_{12}^T} \left( \frac{1}{T_{h}^{TE}} - \frac{1}{T_m} \right)$$  \hspace{1cm} (22)

$$S_{\text{Joule}} = \frac{1}{\sigma R_{22}^T} \left[ \frac{\Delta T}{\sigma m T_m} (M + 1) \right] \left( \frac{1}{T_{h}^{TE}} + \frac{1}{T_m} \right)$$  \hspace{1cm} (23)

Both contributions are constant and set by the operating conditions (hot and cold thermoelectric junction temperatures and thermal demand) and the material properties.

As many equivalent optimal designs could be considered to reach maximal COP, an additional criterion is needed to define a sole optimal thermoelectric module design. Since the cost of THPs is strongly related to the thermoelectric material, we suggest seeking the thermoelectric system requiring the minimum volume $V$ of thermoelectric material. $V$ is defined by $V = N_L L_L A_L$. Combining with (21) gives:

$$V = \frac{L_L^2}{R_{22}^T}$$  \hspace{1cm} (24)

Eq. (24) shows that the thermoelectric material volume is proportional to the square of the leg length (for given $Q_h^*$ and maximum COP). In conclusion, minimizing the leg length minimizes the thermoelectric material volume (and thus its cost) without reducing its performance while the design complies with the optimal ratios. Consequently, the authors propose to set the leg length to a minimum value based on technology feasibility. Considering manufacturing limitations, the minimum leg length $L_L$ has been set to 1 mm.

With respect to Eq. (21), the total legs surface $A_{TE} = N_L^* A_L^*$ (linked to the thermoelectric module footprint) decreases with decreasing leg length. By the way, it tends to increase the thermal flux densities, thus increasing the temperature difference at the thermoelectric junctions (for given thermal resistance and source temperatures) and resulting in a slight increase of the system performance. This second order influence of the thermoelectric design on the global system performance may result in a competition between the operational cost (performance) vs. the investment cost (material cost), not discussed in this paper.

Figure 5 plots the evolution of the maximum COP and the minimum entropy generation vs. the thermal resistance of the heat exchangers, for different leg length. As the curves for different leg length are coincident, it confirms that the maximum COP and the minimum entropy generation are not a function of the leg length (and the corresponding optimal design), but depend only on the thermoelectric properties and the thermal resistances that set the junction temperatures for given source temperatures and thermal demand. As expected, the entropy generation increases as the thermal resistances of the heat exchangers increase. Indeed, as the source temperatures are given, an increase of the thermal resistance results in an increase of the junction temperature difference that decreases the system performances.

For further analysis, both dissipative contributions in the thermoelectric module (conduction and Joule effect) are plotted in Figure 6 vs. the thermal resistance for different leg length (coincident curves).

![Figure 5. Entropy generation rate and COP of the thermoelectric module vs. the thermal resistance for different leg length (coincident curves).](image_url)

![Figure 6. Entropy generation by conduction and Joule effect and their ratio vs the thermal resistance](image_url)
3.4 Whole System Analysis

The optimization of the whole system design has been carried out with the minimization of the whole system entropy generation \( S_{gen}^{tot} \) as the objective function and the thermoelectric module and the heat exchangers designs as optimization variables. The whole system entropy generation \( S_{gen}^{tot} \) is defined as the sum of the entropy generation in each component of the system:

\[
S_{gen}^{tot} = S_{gen}^{hx h} + S_{gen}^{TE} + S_{gen}^{hx c}
\]  

(25)

The thermoelectric module design optimization has been done analytically with respect to the optimal ratio \( R_{2}^{TE} \). While the heat exchanger design optimization has been performed numerically. The optimization procedure has been carried out considering the operating parameter summarized in Table 2 and the variable bounds in Table 3.

![Figure 7. Total entropy generation rate and global COP of the whole system vs the heat exchanger footprint.](Image 319x676 to 547x799)

For further interpretation, the entropy generation contributions of the thermoelectric module and both heat exchangers vs the heat exchanger area are plotted in Figure 8. As expected, irreversibilities in all the components decrease with increasing heat exchanger area, except for the thermoelectric module around the heat exchanger design change. The singularity observed in Figure 7 and 8 for heat exchanger area close to 1.5 m² are due to the heat exchanger design configuration transition (configuration 1 and configuration 2). As this transition is difficult to determine accurately, a “no confidence area” is plotted to highlight this numerical difficulty. Note that configuration 2 (with turbulent flow) helps to reduce significantly the thermal resistance allowing to decrease the thermoelectric module dissipation (compensated by higher heat exchanger dissipations). Figure 8 also points out that the major part of the entropy generation takes place in the thermoelectric module. Moreover it can be observed that the entropy generation in the hot heat exchanger is higher than in the cold one. These results are in agreement with Kaushik [9] and Han [6].

![Figure 8. Entropy generation rates in each component of the system vs the heat exchanger footprint.](Image 63x461 to 291x591)

As the thermoelectric area is almost constant and set by the manufacturing constraints (see section 3.3), the area ratio \( R_{A} = A_{hx}/A_{TE} \) increases almost linearly with the heat exchanger area. In order to comply with the manufacturing constraints, an area ratio of 4 has been chosen to propose realistic heat pump design. This ratio assumes that the heat exchangers footprint equals the thermoelectric module footprint with regularly spread legs, with a spacing distance equal to their width. The corresponding optimal design is given in Table 4. The corresponding design leads to turbulent flow in the channels (configuration 2) and a COP equal to 1.9. The main operative conditions are given in Table 5.

![Table 4. Optimal design of the whole system considering an area ratio \( R_{A} \) equal to 4.](Image 25x25 to 25x25)

<table>
<thead>
<tr>
<th>Design parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area of the heat exchangers (( A_{hx} ))</td>
<td>0.15 m²</td>
</tr>
<tr>
<td>Diameter of the heat exchanger channels (( D ))</td>
<td>4 \times 10^{-3} m</td>
</tr>
<tr>
<td>Length of the heat exchanger channels (( L ))</td>
<td>4.7 m</td>
</tr>
<tr>
<td>Number of heat exchanger channels (( N_{a} ))</td>
<td>4</td>
</tr>
<tr>
<td>Number of the thermoelectric legs (( N_{L} ))</td>
<td>2046</td>
</tr>
<tr>
<td>Area of the thermoelectric legs (( A_{TE} ))</td>
<td>1.86 \times 10^{-5} m²</td>
</tr>
</tbody>
</table>

![Table 5. Optimal internal operating conditions of the whole system considering an area ratio \( R_{A} \) equal to 4.](Image 25x25 to 25x25)

<table>
<thead>
<tr>
<th>Design parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>COP of the whole system</td>
<td>1.90</td>
</tr>
<tr>
<td>Total entropy generation (( S_{gen}^{tot} ))</td>
<td>4.78 W.K⁻¹</td>
</tr>
<tr>
<td>Thermal power exchanged at the cold side of the thermoelectric module (( Q_{c} ))</td>
<td>1473.4 W</td>
</tr>
<tr>
<td>Electrical power consumed by the thermoelectric module (( W_{TE} ))</td>
<td>1506.5 W</td>
</tr>
<tr>
<td>Electric current in the thermoelectric legs (( I_{L} ))</td>
<td>29.1 A</td>
</tr>
<tr>
<td>Reynolds number of the fluid in the heat exchanger channels (( Re ))</td>
<td>11318</td>
</tr>
<tr>
<td>Temperature of the hot heat exchanger (( T_{h} ))</td>
<td>3041 K</td>
</tr>
<tr>
<td>Temperature of the cold heat exchanger (( T_{c} ))</td>
<td>277.7 K</td>
</tr>
</tbody>
</table>
The entropy flows in the system for the specific design considered \((R_s = 4)\) are plotted in Figure 9, where the entropy generation sources are highlighted in red. This diagram points out that the joule effect in the most dissipative phenomena. It also highlights that for that optimal design the irreversibilities due to heat transfer are higher than those due to viscous dissipation for the hot heat exchanger, whereas it is the inverse for the cold one.

![Figure 9. Entropy flow through the system for optimal design considering an area ratio \(R_s\) equal to 4.](image)

4. Conclusion

This paper aims to propose a design strategy for water/water thermoelectric heat pump with high performance based on entropy generation minimization. The system studied is composed of a thermoelectric module sandwiched between the hot and cold heat exchangers to transfer heat from the sources to the thermoelectric junctions. The design optimization variables considered in this study are the number of thermoelectric legs \(N_L\), their length \(L_L\) and section \(A_L\) for the thermoelectric module, and the number of channel \(N\), their diameter \(D\) and their length \(L\) for both heat exchangers. This study assumes that the operating conditions, such as the inlet/outlet temperatures in both heat exchangers and the thermal demand are defined. Moreover, the thermoelectric material and fluid properties have to be known. A second law analysis allows distinguishing and quantifying each dissipative phenomenon occurring in the system: Joule effect and thermal conduction in the thermoelectric module, and viscous friction and thermal transfer in both heat exchangers. Optimal design resulting from entropy generation minimization procedure is then discussed.

On the one hand, three optimal thermoelectric design ratios, giving identical thermoelectric performances, are derived analytically. These design ratios link the thermoelectric module design (length, section and number) to the operating electric current, for given junction temperatures and material properties. The authors propose to reduce as possible the leg length, with respect to the manufacturing process, as it is directly linked (for optimal designs) to the total thermoelectric material volume (and thus to the investment cost).

On the other hand, the heat exchanger optimization leads to two distinct configurations, depending on the thermal power density to exchange. For high heat exchanger footprint (i.e. low thermal power density), the optimal design is reached for laminar flow (configuration 1), with a high number of channels of minimum diameter and length. Inversely, the optimal heat configuration switches to a turbulent flow design (configuration 2) for smaller area (i.e. higher thermal power density), with a low number of channels of high diameter and length. The design switch condition results from the competition of both entropy generation contributions (heat transfer and viscous dissipation) and is highly dependent of the variables range considered.

Finally, when considering the whole system, entropy generation in the thermoelectric module highly dominates those in the heat exchangers, mainly because of Joule effect. However, the heat exchangers design highly impacts the global THP performances, as the thermoelectric junction temperatures depends on the heat exchanger design. By the way, the system footprint slightly impacts the optimized system performance. An area ratio between the heat exchanger footprint and the thermoelectric module area of 4 is selected to comply with integration constraints. In this case, the optimal design corresponds to turbulent flow in the heat exchangers, as the thermal power density remains high. Under that design, the system COP reaches a value of 1.9 for the given operating conditions.

### Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>area, (\text{m}^2)</td>
</tr>
<tr>
<td>(C_p)</td>
<td>specific heat, (\text{J/(kg K)})</td>
</tr>
<tr>
<td>(COP)</td>
<td>coefficient of performance, -</td>
</tr>
<tr>
<td>(D)</td>
<td>channel diameter, (\text{m})</td>
</tr>
<tr>
<td>(f)</td>
<td>friction factor, -</td>
</tr>
<tr>
<td>(h)</td>
<td>heat transfer coefficient, (\text{W/(m}^2\text{ K)})</td>
</tr>
<tr>
<td>(K)</td>
<td>thermal conductance, (\text{W/K})</td>
</tr>
<tr>
<td>(L)</td>
<td>length, (\text{m})</td>
</tr>
<tr>
<td>(m)</td>
<td>mass flow rate, (\text{kg/s})</td>
</tr>
<tr>
<td>(N)</td>
<td>Number of channels/legs, -</td>
</tr>
<tr>
<td>(Nu)</td>
<td>Nusselt number, -</td>
</tr>
<tr>
<td>(\Delta P)</td>
<td>pressure drop, (\text{Pa})</td>
</tr>
<tr>
<td>(P)</td>
<td>pressure, (\text{Pa})</td>
</tr>
<tr>
<td>(Pr)</td>
<td>Prandtl number, -</td>
</tr>
<tr>
<td>(Q)</td>
<td>thermal power, (\text{W})</td>
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<tr>
<td>(dQ)</td>
<td>thermal power density, (\text{W/m}^2)</td>
</tr>
<tr>
<td>(R)</td>
<td>electric resistance, (\Omega)</td>
</tr>
<tr>
<td>(R_s)</td>
<td>thermal resistance, (\text{W/K})</td>
</tr>
<tr>
<td>(Re)</td>
<td>Reynolds number, -</td>
</tr>
<tr>
<td>(S)</td>
<td>entropy flow, (\text{W/K})</td>
</tr>
<tr>
<td>(S_{gen})</td>
<td>entropy generation rate, (\text{W/K})</td>
</tr>
<tr>
<td>(T)</td>
<td>temperature, (\text{K})</td>
</tr>
<tr>
<td>(v)</td>
<td>velocity, (\text{m/s})</td>
</tr>
<tr>
<td>(\Delta V)</td>
<td>electrical voltage, (\text{V})</td>
</tr>
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### Greek Symbols

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>(\alpha)</td>
<td>Seebeck coefficient, (\text{V/K})</td>
</tr>
<tr>
<td>(\lambda)</td>
<td>thermal conductivity, (\text{W/(m K)})</td>
</tr>
<tr>
<td>(\mu)</td>
<td>viscosity, (\text{Pa.s})</td>
</tr>
<tr>
<td>(\rho)</td>
<td>fluid density, (\text{kg/m}^3)</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>electric resistivity, (\Omega\cdot\text{m})</td>
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### Subscripts and Superscripts

<table>
<thead>
<tr>
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<tr>
<td>(c)</td>
<td>cold</td>
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<tr>
<td>(ch)</td>
<td>channel</td>
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<tr>
<td>Cond</td>
<td>conduction</td>
</tr>
<tr>
<td>(ex)</td>
<td>exchange</td>
</tr>
<tr>
<td>(h)</td>
<td>hot</td>
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</table>
heat exchanger
inlet
Joule effect
thermoelectric legs
outlet
pressure drop
thermoelectric module
thermal transfer

References