Investigation on Optimal Thermohydraulic Performance of a Solar Air Heater Having Arc Shaped Wire Rib Roughness on Absorber Plate

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Abstract

This paper presents an investigation on optimal thermohydraulic performance of a solar air heater having arc-shaped wire rib roughness on underside of the absorber plate. The use of artificial roughness on the absorber plate of a solar air heater causes enhancement in rate of the heat transfer between absorber plate and flowing air, by creating turbulence, however, there is substantial increase in the frictional resistance which leads to more pumping power consumption. The effective efficiency is evaluated on the basis of useful thermal energy gain minus equivalent thermal energy to the power required to propel the air through solar air heaters. A mathematical model and procedure for prediction of effective efficiency of an arc shaped wire rib roughened absorber plate solar air heater has been presented and the effects of roughness and various geometrical and operating parameters on effective efficiency have been worked out. The effective efficiency of the roughened absorber plate solar air heater has also been compared with smooth absorber plate solar air heater operating under similar conditions. It is observed that effective efficiency of a roughened solar air heater has an optimum value corresponding to a certain Reynolds number and specific values of operating and geometrical conditions. A correlation between system and operating parameters that delivers optimum effective efficiency has been developed and presented, which can be used for optimal design of such systems.

Keywords: Effective efficiency; artificial roughness; solar air heater; arc shaped wire rib roughness.

1. Introduction

A conventional solar air heater, Figure 1(a), is essentially a flat plate collector consisting of an absorber plate, a transparent cover at the top and insulation at the bottom and on the sides. The incident solar radiation is absorbed and converted by the absorber plate into thermal energy which is utilized for heating air passing through it. The heated air can be used in many low to moderate temperature applications [1]. The major disadvantages of a solar air heater is its lower thermal efficiency due to low heat transfer coefficient between absorber plate and flowing air and the need for handling large volumes of air due to its low density which results in greater pumping power requirement to propel the air through the heater duct. The heat transfer coefficient between absorber plate and the flowing air and thereby the thermal efficiency of a conventional solar air heater has been considerably improved by adopting several methods [2,3].

The use of artificial roughness of different geometries on absorber plate surface results in enhanced rate of heat transfer between flowing air in solar air heater ducts, as it breaks the laminar sub-layer which causes successful reattachment of breaking sublayer flows. However, this roughness results in an increase in friction losses and hence, greater power requirement by fan or blower. To account the thermohydraulic performance of a solar air heater, effective efficiency or thermohydraulic efficiency is determined that includes both the terms: useful thermal energy gain and pumping power expended. The emphasis was given by the researchers in this direction, who suggested use of the roughness elements that delivered maximum thermal efficiency with minimum pumping power requirement. Prasad and Saini [4] investigated fully developed turbulent flow in a solar air heater duct with transverse wire rib roughness on the absorber plate. They applied wall similarity law and reproduced the heat transfer and friction factor correlations as developed earlier by [5] and [6]. Karmare and Tikekar [7] investigated experimentally the optimal thermohydraulic performance of solar air heaters with metal rib grits roughness and presented correlation for Nusselt number and friction factor for optimal performance condition. Karwa et al. [8] used chamfered rib roughness on the absorber plate and they investigated thermohydraulic efficiency on the basis of their experimental work. They reported that the solar air heaters having higher relative roughness height of the roughness elements yields a better performance. Bhushan et al. [9] did experimental investigation to analyze the effect of artificial roughness on heat transfer and friction in solar air heater duct having protruded surface as roughness on the absorber plate and predicted the values of maximum enhancement of Nusselt number and friction factor. Singh et al. [10] carried out their investigation on thermohydraulic performance of solar air heater duct having varying flow-attack-angle in V-down rib with gap roughness geometry. Prasad and Saini [11] studied the performance of solar air heater having transverse rib on the underside of the absorber plate and presented the results in terms of roughness height (e), thickness of laminar sublayer (δ) and roughness Reynolds number (Re*) for optimum performance. The effect of transverse wire roughened absorber plate on the heat and fluid flow...
characteristics through the solar air heater ducts was investigated experimentally by Verma and Prasad [12]. They optimized the results on the basis of method suggested by [13]. Solar air heater having transverse rib was optimized by [12] through their experimental work on the basis of method given by [13]. Prasad et al. [14, 15] optimized the thermohydraulic performance of solar air heater with three sided transverse rib roughness and developed the correlations for Stanton number and friction factor based on the approaches used by [5]. Gupta et al. [16] used continuous inclined ribs having angle of attack of 60° and they analytically optimized the thermohydraulic performance on the basis of methods adopted by [17]. They also derived an empirical relationship that relates the system and operating parameters to yield optimum design conditions. As implied from the discussion on the effect of rib roughness, the heat transfer coefficient enhancement is also accompanied with an increase in the friction factor. Thus, an appropriate way to evaluate performance, in case of solar air heaters with artificially roughened absorber plate, is to take both useful heat collection rate and pumping power requirement into account, i.e. to carry out a thermohydraulic performance or effective efficiency evaluation.

The objective of the present investigation is to evaluate the effective efficiency of solar air heater, with arc shaped wire rib roughness on absorber plate. The length (L), width (W), duct height (H), wind speed (V), inlet air temperature (Ti) and atmospheric air temperature (Ta) were taken as constant, while relative roughness height (e/D), relative angle of attack (α/90), insolation (I) and mass flow rate in terms of Reynolds number (Re) have been considered as variable parameters. Furthermore, an empirical correlation which delivered optimal effective efficiency for selected parameters has to be developed. Typical values of system and operating parameters, used in the present investigation, have been shown in Table 1.

2. System Modeling
2.1. Analysis of solar air heater

The effective efficiency of solar air heaters is defined as the ratio of net thermal energy gain to the input thermal energy and is given by [17]:

$$\eta_{eff} = \frac{Q_u - P_w/\zeta}{IA}$$

(1)

where $\zeta$ is a factor that accounts for conversion of mechanical energy or pumping power into equivalent thermal energy and is given by

$$\zeta = \eta_f \eta_{fa} \eta_{h_b}$$

(1a)

The rate of useful thermal energy gain is calculated from the equation:

$$Q_u = F_p \left[ \frac{I(\tau \alpha_e) - U_1(T_f - T_a)}{2} \right] A_c$$

(2)

Where, $T_f = (T_o + T_i)/2$ and

$$F_p = \frac{h}{(h+U_1)}$$

(3)

Eq. (2) can be expressed in a more usable form by introducing the term ‘collector heat removal factor’ FR, defined as the ratio of actual useful heat collection rate to the useful heat collection rate attainable with absorber plate surface temperature at the inlet air temperature,

$$Q_u = F_R \left[ \frac{I(\tau \alpha_e) - U_1(T_f - T_a)}{2} \right] A_c$$

(4a)

In a particular case, when the solar air heaters draw air at ambient temperature (i.e. $T_i = T_a$), the useful heat gain of a solar air heater is given by [1]:

$$Q_u = F_o \left[ \frac{I(\tau \alpha_e) - U_1(T_f - T_a)}{2} \right] A_c$$

(4b)

where $F_o$ is the heat removal factor referred to outlet air temperature and is given by

$$F_o = \frac{m C_p}{U_o A_c} \left[ \frac{U_o A_c}{\alpha C_p (T_o - T_f)} - 1 \right]$$

(4c)

The rate of useful energy gain in a solar air heater may also be calculated from the equations:

$$Q_u = m C_p (T_o - T_f)$$

(5)
The mechanical power expended is given by the expression:

\[ P_m = \frac{m(\Delta P)}{\rho} \]  

where,  

\[ \Delta P = \frac{4fLV^2}{2D} \]  

where the hydraulic diameter \( D \) of the duct is given by

\[ D = \frac{2(W \times H)}{(W + H)} \]  

The thermal efficiency of solar air heater is given as:

\[ \eta_a = \frac{Q_u}{IA} \]  

In order to find the value of heat transfer coefficient for smooth solar air heater duct, Dittus –Boelter equation was used [18]:

\[ Nu_s = 0.024(Re)^{0.8}(Pr)^{0.4} \]  

From which the heat transfer coefficient between the absorber plate and the air flowing over it \( h \), is calculated as

\[ h = \frac{K \cdot Nu_s}{D} \]  

Heat transfer coefficient for arc shaped roughened solar air heater duct can be calculated using the equation, given below [19]:

\[ Nu_e = 0.001047(Re)^{1.3186}(Pr)^{0.1198} \]  

The friction factor \( f_s \) for smooth solar air heater is given by Blasius equation [18]:

\[ f_s = 0.085(Re)^{-0.25} \]  

The friction factor \( f_r \) for roughened solar air heater is given by [19]:

\[ f_r = 0.14408(Re)^{-0.17013}(Pr)^{0.1765}(\alpha/90)^{0.1185} \]  

The overall heat loss coefficient \( U_L \) of a solar air heater is the sum of the top loss coefficient \( U_t \), back loss coefficient \( U_b \), and side loss coefficient \( U_s \) and is given by

\[ U_L = U_t + U_s + U_b \]  

The top loss coefficient, \( U_t \) is evaluated by using the equation as proposed by [20].

\[ U_t = \left[ \frac{M}{c} \left( \frac{T_p - T_a}{T_p} \right)^{0.252} + \frac{1}{h_v} \right]^{-1} + \left[ \frac{\sigma(T_p^4 + T_a^4)(T_p + T_a)}{1 + 0.0425M(1 - \varepsilon)} + \frac{2M + f^+ - 1}{M} \right] \]  

where

\[ c = 204.429((\cos \beta)^{0.252} / L_v^{0.24}) \]  

\[ f^+ = (9/h_v) - (30/h_v^2)(T_p/316.9)(1 + 0.091M) \]  

and

\[ h_v = 5.7 + 3.8V_w \]  

The back heat loss coefficient \( U_b \) is calculated using the relation given below:

\[ U_b = \frac{K_i}{\delta_i} \]  

The side heat loss coefficient \( U_s \) is calculated using the relation given by:

\[ U_s = \frac{(L+W) \cdot H \cdot K_i}{L \cdot W \cdot \delta_i} \]  

Roughness Reynolds number \( Re^+ \) is calculated using the relation given below [21]:

\[ Re^+ = (e/D) \sqrt{\frac{f}{2}} \cdot Re \]  

Reynolds number \( Re \) is calculated by:

\[ Re = \frac{G \cdot D}{\mu} \]  

where mass velocity of air \( G \) is calculated using the relation given below:

\[ G = m/(W \cdot H) \]  

and mass flow rate of air \( m \) is calculated by:

\[ m = \frac{C_p}{(Qu \cdot \Delta T)} \]  

The Enhancement ratio \( ER \) is the ratio of effective efficiencies of roughened solar air heater to the smooth absorber plate solar air heater

\[ ER = \frac{(Effective \, efficiency) \, Rough}{(Effective \, efficiency) \, Smooth} \]  

Nikursade [21] defined and developed the theory to categorise the fluid flow for the roughened surface into three regions (laminar, transition and fully developed turbulent)
based on the values of Roughness Reynolds number (Re*), which is calculated by the Eq. (22). The three regions are

\[
\begin{align*}
0 &< Re^* < 5 & \text{Laminar region} \\
5 &\leq Re^* \leq 70 & \text{Transition region} \\
Re^* &> 70 & \text{Fully developed turbulent region}
\end{align*}
\]

(27) (28) (29)

3. System and operating parameters for roughened solar air heater

The absorber plate of the roughened solar air heater is simply a thin metallic sheet that consists of arc shaped wire as the roughness elements fixed on its back side as shown in Figure 1 (b).

In order to evaluate thermohydraulic efficiency of the roughened solar air heater, values of system and operating parameters were selected as given in Table 1.

Figure 1(b). Arc shape wire rib roughness geometry on absorber plate.

4. Results and discussion

By employing mathematical model for solar air heaters and their system and operating parameters, various design plots have been presented (Figures 2 – 13). The performance evaluation has also been carried out to explore the effect of relative roughness height (e/D), relative angle of attack (\(\alpha/90\)), and solar insolation (I) on thermal efficiency and effective efficiency for arc shaped roughened and smooth plate solar air heaters.

Figure 2 (a) shows the variation of useful heat gain (\(Q_u\)) and the power requirement of the fan for arc shaped wire rib roughened solar air heater as a function of Reynolds number (Re). It is observed that the rate of increase of useful energy gain is comparatively higher for lower range of Reynolds number (Re < 15,000 approx.) whereas this rate of increase is lower at higher range of Reynolds number. The rate of increase of power consumption is low for lower range of Reynolds number (Re < 15,000 approx.) and increases comparatively with faster rate in higher range of Reynolds number. This variation indicates the trend as expected.

Figure 2 (b) shows the comparison of variations of useful heat gain (Qu) and the power requirement for arc shaped wire rib roughened and smooth absorber plate solar air heaters with Reynolds number (Re). It is clear from Figure 2(b) that the rate of increase of useful energy gain is relatively higher at low range of Reynolds number, whereas it is a bit lower at higher range of Reynolds number. But pumping power consumption at the lower Reynolds number (Re < 12,000 approx.) remains almost same for both the smooth and rough solar air heaters, and after this value of Reynolds number it increases rapidly, however the power consumption is more than that of the smooth solar air heaters in this range of Reynolds number. The power consumption does not exceed the rate of useful energy gain, i.e. the net energy gain rate is positive and it is also clear that at higher Reynolds number, the rate of useful energy collected becomes almost constant but the power consumption rises steeply.

Figure 2(a). Energy balance for roughened solar air heater
(b) Comparison of energy-balance for smooth and roughened solar air heater.

4.1. Effect of relative roughness height

Variation of effective efficiency (\(\eta_{eff}\)) with Reynolds number (Re = 2000-25,000), for four different values 0.021, 0.029, 0.036, and 0.042 of relative roughness height (e/D) and insolation (I) of 1000 W/m\(^2\) and 500 W/m\(^2\) have been shown in Figures 3 (a) and (b) respectively. The values of, relative angle of attack (\(\alpha/90\)), and relative roughness pitch (P/e), have been taken constant at 0.33 and 10 respectively in these plots.

It is observed from Figure 3 (a) that as the Reynolds number increases, the effective efficiency increases, attains a maximum value at certain Reynolds number, and then reduces with further increase in Reynolds number. At higher value of e/D the effective efficiency is higher and at lower value of e/D it is lower. Similar trends of variations of effective efficiency are obtained for both values of insolation (I) however, at lower value of insolation (I= 500 W/m\(^2\)) the effective efficiency, after attaining its maximum value, decreases with faster rate than for I= 1000 W/m\(^2\). Thus, there exists an optimum value of effective efficiency for a given roughness configuration. This effect shows that the Reynolds number is a strong parameter that affects the pumping power and useful thermal energy gain, thereby affecting the effective efficiency.

In order to investigate the behavior of relative roughness height (e/D) on the effective efficiency of the collector, the parameters \(\alpha/90\), P/e, and I have been chosen fixed as shown in Figure 3 (b). It is observed that the best thermohydraulic performance is at e/D = 0.042 at Reynolds number value of 15,200 as indicated in Figure 3 (b). This behavior is probably
due to the fact that a higher height of roughness, offer more turbulence to the flow of air in the duct, resulting in higher rate of heat transfer as compared to smaller height of roughness.

**4.2. Effect of relative angle of attack**

Figure 4 (a) and (b) show the variation of effective efficiency ($\eta_{eff}$) with Reynolds number (Re) for different values of relative angle of attack ($\alpha/90$). The fixed parameters, relative roughness height ($e/D$), and relative roughness pitch ($P/e$) were taken as 0.042 and 10 respectively.

It is found from Figure 4 (a) that as the Reynolds number increases, the effective efficiency increases first up to a certain maximum value of Reynolds number, and after that decreases with further increase in Reynolds number. There exists an optimum value of effective efficiency for a given roughness configuration. This effect shows that the Reynolds number is a strong parameter that affects the pumping power and thermal energy gain, thereby affecting the effective efficiency.

In order to investigate the behavior of relative angle of attack ($\alpha/90$) on the effective efficiency performance of the collector, the parameters $e/D$, $P/e$ and $I$ have been chosen as shown in Fig. 4 (b). It is observed that the lowest value of angle of attack $\alpha/90=0.33$ results in highest thermohydraulic efficiency whereas at the highest value of $\alpha/90=0.66$ it is the lowest for all values of Reynolds number $Re < 20,000$ and $Re < 18,000$ as indicated in Figure 4 (a) and 4 (b) respectively itself.
It can be noticed from Figure 5 (a) and (b) that for a particular value of insolation, effective efficiency increases and goes up to a maximum at a certain value of Reynolds number and after that it starts decreasing. The point corresponding to the maximum value of effective efficiency is found to shift to higher side of Reynolds number as the value of insolation increases. This leads to the fact that the flow rate of the air through the air heater duct should be adjusted to a Reynolds number such that the maximum effective efficiency for that value of insolation can be attained. Effective efficiency increases with increase in insolation values. This appears due to increase in the absorber plate temperature with increase in insolation and thus leading to higher heat transfer to the air, required pumping power remaining unchanged.

Figure 6 (a) and (b) show the variation of useful energy collected (Qu) and the thermal energy equivalent to the power consumed (Pm/Q) as a function of Reynolds number for two values of insolation 1200 W/m2 and 700 W/m2.

From Figures 6 (a) it is revealed that Qnet increases in the lower range of Reynolds number, archives a maximum value and thereafter decreases. The rate of increase of Qnet is faster at higher value of insolation (I =1200 W/m2) than that of the lower value of insolation (I =700 W/m2). On the other hand, the value of (Pm/Q) increases with a very slow rate in the lower range of Reynolds number (Re < 14,000) but increases rapidly in the higher range of Reynolds number, as can be seen in Figure 6 (b). It is also observed that the maximum point of net energy gain moves towards the lower Reynolds number with decrease in the insolation. This seems to happen due to reduction in absorber plate temperature owing to low value of insolation and also due to lower mass flow rate of air that leads to decrease in rate of heat transfer to air in the duct, while the pumping power expended remains the same.

4.4. Effect of Reynolds number

Figure 7 shows the variation of effective efficiency with insolation (I) for different values of Reynolds number for roughened solar air heater for constant values of e/D, P/e, and α/90. The effective efficiency is found to increase with insolation for all values of Reynolds number. However, in the range of Reynolds number 17,100 to 3800 it increases more rapidly as compared to other Reynolds number (Re < 17,000) as shown in Figure 7.

4.5. Thermal and thermohydraulic efficiency comparison

Figures 8 (a) and (b) shows the thermal and effective efficiencies of arc-shaped rough and smooth plate solar air heaters as a function of Reynolds number.

In Figure 8 (a) the curves have been drawn for roughened solar air heater and is observed that the thermal efficiency increases continuously with Reynolds number, whereas the effective efficiency first increases, attains a maximum value and thereafter decreases with increase in Reynolds number. This trend of variation of effective efficiency indicates that the net thermal energy gain goes on decreasing after attaining its maximum value owing to increase in the equivalent energy gain against pumping power expended. Further, it can be attributed that there exists an optimum operating condition for a given configuration of roughness geometry at which the effective efficiency is maximum for a particular Reynolds number.

Figure 8 (b) shows the comparison for roughened and smooth solar air heaters as a function of roughness Reynolds number (Re*). It can be observed that the thermal efficiency of rough as well as smooth solar air heaters increases continuously for all values of roughness Reynolds number (Re*). On the other hand effective efficiency of both solar air heaters increases in the transition region (5 ≤ Re* ≤ 70). The effective efficiency of roughened solar air heater attains its maximum value at roughness Reynolds number (Re*) of 80 (approx.) and after that it starts decreasing with increase in roughness Reynolds number, in case of roughened solar air heater. The effective efficiency of roughened solar air heater in fully developed turbulent flow region (Re* > 80), decreases, however it can be concluded that the use of roughness in the solar air heater is more beneficial as compared to smooth solar air heater up to a roughness Reynolds number of (Re* = 125 ) from effective efficiency point of view.
4.6. Variation of Enhancement ratio $E_R$

The variation of Enhancement ratio $E_R$ of effective efficiencies of roughened solar air heater to that of smooth solar air heater with the Reynolds number for different relative roughness heights (e/D) at insolation $I = 1000 \text{ W/m}^2$, relative roughness pitch $P/e = 10$ and relative angle of attack $\alpha/90 = 0.33$, has been shown in Figure 9. It is observed that the ratio ($E_R$) is highest for highest value of e/D and is the lowest for lowest value of e/D, however this trend of variation for all values of e/D is the same. The Reynolds number corresponding to the maximum $E_R$ shifts towards lower values of Reynolds number as e/D increases.

4.7. Optimum design condition

From the above results it is observed that the point of maximum effective efficiency is a strong function of roughness parameters and insolation. The Reynolds number corresponding to optimum conditions also changes with roughness parameters and insolation. The performance plots for the roughened solar air heater have been prepared for a range of relative roughness height e/D = 0.021 - 0.042, roughness heights e/D, have been plotted. It is found that effective efficiencies Enhancement ratio ($E_R$) is the highest for the lowest value of angle of attack $\alpha/90$ whereas, it is the lowest for the highest value of $\alpha/90$ and the nature of variation is the same for all values of $\alpha/90$. There is an optimum value of ratio $E_R$ corresponding to a particular value of Reynolds number.

It is also observed that a roughened solar air heater operating at lower Reynolds number (Re < 3000) is not thermohydraulically efficient than the smooth absorber plate solar air heater. The effective efficiency Enhancement ratio $E_R$ first increases, attains a maximum value and then decreases with increase in Reynolds number. This trend of variation is observed for all values of $\alpha/90$. However, a critical value of Enhancement ratio of effective efficiencies ($E_R$), corresponding to a particular Reynolds number (Re = 19,000) is obtained at which the variation in relative angle of attack $\alpha/90$ has no effect on enhancement ratio of effective efficiencies.

From the above results it is observed that the point of maximum effective efficiency is a strong function of roughness parameters and insolation. The Reynolds number corresponding to optimum conditions also changes with roughness parameters and insolation. The performance plots for the roughened solar air heater have been prepared for a range of relative roughness height e/D = 0.021 - 0.042,
Reynolds number \( Re = 2000 – 25,000 \) and insolation \( I = 500 – 1200 \) W/m². Using the values of Reynolds numbers that correspond to the maximum effective efficiency for corresponding values of relative roughness height \( e/D \) and insolation \( I \), the variation of relative roughness height \( e/D \) with Reynolds number \( Re \) for different values of insolation for optimum conditions has been shown in Figure 11. It is found from this Figure, that with the increase in insolation, for a given relative roughness of height \( e/D \), Reynolds number that shows the maximum effective efficiencies, increases.

Figure 12 shows a plot exhibiting the relationship between the Reynolds number \( Re \) and insolation \( I \) which shows the combination of relative roughness height \( e/D \), Reynolds number \( Re \) and insolation \( I \) for optimum operating conditions of the roughened solar air heater.

### Table 2: Optimum conditions

<table>
<thead>
<tr>
<th>( I, ) W/m²</th>
<th>( e/D=0.021 )</th>
<th>( e/D=0.029 )</th>
<th>( e/D=0.036 )</th>
<th>( e/D=0.042 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>Re=17710</td>
<td>Re=17450</td>
<td>Re=17230</td>
<td>Re=17030</td>
</tr>
<tr>
<td>110</td>
<td>Re=17590</td>
<td>Re=17330</td>
<td>Re=17110</td>
<td>Re=16920</td>
</tr>
<tr>
<td>100</td>
<td>Re=17450</td>
<td>Re=17200</td>
<td>Re=16980</td>
<td>Re=16800</td>
</tr>
<tr>
<td>700</td>
<td>Re=17050</td>
<td>Re=16800</td>
<td>Re=16600</td>
<td>Re=16400</td>
</tr>
<tr>
<td>500</td>
<td>Re=16650</td>
<td>Re=16400</td>
<td>Re=16200</td>
<td>Re=16000</td>
</tr>
</tbody>
</table>

The following equation, that relates the system and operating parameters, was obtained using regression method.

\[
Re_{\text{optimal}} = 8.808 \times 10^{3 \, (I^{0.066} \times (e/D)^{-0.056})} \]

(30)

This equation correlates the data (From Table-2) for optimum conditions with a regression coefficient, \( R^2 = -0.05466 \).

Figure 12. Reynolds number \( (Re) \) verses Insolation \( (I) \).

By adopting the same procedure as above, an equation, that relates the operating parameters and the roughness parameter \( \alpha/90 \), for constant values of the parameters \( e/D \) and \( P/e \) was obtained as given below.

\[
Re_{\text{optimal}} = 10.158 \times 10^{4 \, (I^{0.0701} \times (\alpha/90)^{-0.0851})} \]

(31)

Figure 13. Plots of \( Re/I^{0.4625} \) verses relative roughness height \( (e/D) \).

5. Comparison of effective efficiencies of arc shaped and other roughness geometries

Figure 14 shows the comparison of effective efficiencies of arc shaped wire rib roughened solar air heater, obtained in the present work with effective efficiencies of solar air heaters having other roughness geometries, for a common roughness parameters and operating parameters, using the respective correlations as given in Table-3. It is observed that every roughness geometry corresponds to an optimum value of effective efficiency of solar air heater corresponding to a particular Reynolds number. The solar air heater with W-shape wire rib roughness \([22]\) exhibits the maximum efficiency for Reynolds number \( Re < 8,000 \) over all the roughness geometries, however, it has the lower effective efficiency than the arc shaped wire rib roughness (Present work) in the higher range of Reynolds number \( Re > 8,000 \). All other roughness geometries have lower effective efficiency as compared to arc shaped wire rib roughened solar air heater for all values of Reynolds number.

Figure 14. Comparison of effective efficiency for arc-shaped wire rib roughness with other roughness geometries of roughened solar air heaters.

5. Conclusions
The following are the conclusions of the present investigation:

- The efficiency of the arc-shaped wire rib roughened solar air heater is higher than other roughness geometries for all values of Reynolds number.
- All other roughness geometries have lower effective efficiency as compared to arc shaped wire rib roughened solar air heater for all values of Reynolds number.
### Table 3. Comparison of effective efficiency of solar air heater having arc-shaped wire rib roughness with other roughness geometries on the absorber plate.

<table>
<thead>
<tr>
<th>Roughness Geometry and Investigators</th>
<th>Roughness Parameters for evaluation</th>
<th>Nusselt number (Nu)</th>
<th>Friction factor (f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small diameter protrusion wire ribs</td>
<td>e/D = 0.042, P/e = 10, Re = 2000-25,000, W/H = 10, l = 1000 W/m²</td>
<td>Nu = 0.000824(e/D)^0.176 (W/H)^0.264 Re^0.82</td>
<td>f = 0.06412(e/D)^0.509 (W/H)^0.257 Re^0.185</td>
</tr>
<tr>
<td>Gupta et al. [23]</td>
<td></td>
<td>For Re ≤ 35</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Nu = 0.000307(e/D)^0.499 (W/H)^0.245 Re^0.812</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>For Re ≥ 35</td>
<td></td>
</tr>
<tr>
<td>Inclined continuous wire ribs</td>
<td>e/D = 0.042, P/e = 10, Re = 2000-25,000, α = 60°, W/H = 10, l = 1000 W/m²</td>
<td>Nu = 0.00244(e/D)^0.009 (W/H)^0.009 Re^1.04 exp[−0.04(1−α/60)^2]</td>
<td>f = 0.19111(e/D)^0.916 (W/H)^0.009 Re^0.105 exp[−0.0993(1−α/60)^2]</td>
</tr>
<tr>
<td>Gupta et al. [16]</td>
<td></td>
<td>For Re ≤ 35</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Nu = 0.0071(e/D)^-0.24 (W/H)^-0.028 Re^0.866 exp[−0.475(1−α/60)^2]</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>For Re ≥ 35</td>
<td></td>
</tr>
<tr>
<td>Combination of Inclined and Transverse ribs</td>
<td>e/D = 0.042, P/e = 10, Re = 2000-25,000, l = 1000 W/m²</td>
<td>Nu = 0.0006 Re^1.213 (P/e)^0.004</td>
<td>f = 1.0858 Re^-0.265 (P/e)^0.0114</td>
</tr>
<tr>
<td>Varun et al. [24]</td>
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<tr>
<td>W-shape wire ribs</td>
<td>e/D = 0.042, P/e = 10, Re = 2000-25,000, α = 60°, l = 1000 W/m²</td>
<td>Nu = 0.0613 Re^0.9079 (e/D)^0.4487 (α/60)^0.131 exp[−0.5307(ln(α/60))^2]</td>
<td>f = 0.6182 Re^-0.2254 (e/D)^0.4823 (α/60)^0.1017 exp[−0.28(ln(α/60))^2]</td>
</tr>
<tr>
<td>Lanjewar et al. [22]</td>
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<tr>
<td>Metal grit ribs</td>
<td>e/D = 0.042, P/e = 10, Re = 2000-25,000, l/s = 1.72, l = 1000 W/m²</td>
<td>Nu = 2.4×10^-3, Re^1.3 (e/D)^0.42 (l/s)^-0.146 (P/e)^-0.27</td>
<td>f = 15.55 Re^-0.263 (e/D)^0.91 (l/s)^-0.27 (P/e)^0.51</td>
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<tr>
<td>Karmare and Tikekar [25]</td>
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<tr>
<td>Arc shaped wire rib roughness</td>
<td>e/D = 0.042, P/e = 10, Re = 2000-25,000, α/90 = 0.33, l = 1000 W/m²</td>
<td>Nu = 0.001047(Re)^1.3196 (e/D)^0.3772 (α/90)^-0.1198</td>
<td>f = 0.14408(Re)^-0.17203 (e/D)^0.1105</td>
</tr>
<tr>
<td>(Present work)</td>
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</tbody>
</table>

Int. J. of Thermodynamics (IJoT) Vol. 19 (No. 4) / 222
1. The rate of increase of useful heat gain is comparatively higher for roughened solar air heater for the lower range of Reynolds number ($Re < 15,000$) whereas this rate of increase is lower at higher range of Reynolds number. The heat gain of roughened solar air heater is having greater values as compared to smooth solar air heater for all values of Reynolds number.

2. There is a significant increase in the rate of power consumption for both solar air heaters for higher range of Reynolds number ($Re > 12,000$), however for the lower range of Reynolds number ($Re < 12,000$) power consumption remains same for both solar air heaters.

3. Thermal efficiency and effective efficiency increase significantly up to transition region. Thermal efficiency increases continuously for roughened and smooth solar air heater for all the values of Reynolds numbers, but effective efficiency of roughened solar air heater starts decreasing after attaining some optimum value corresponding to a particular Roughness Reynolds number.

4. For each value of relative roughness height ($e/D$) and also relative angle of attack ($\alpha/90$), the effective efficiency is having maximum value for particular value of Reynolds number. The values of effective efficiency increases with increase in the value of Insolation ($I$)

5. An equation, that relates the system and operating parameters, is obtained for the optimum conditions for arc shaped wire rib roughened solar air heater as given below:

$$Re_{\text{optima}} = 8.808 \times 10^3 \left[ (I)^{0.068} \times (e/D)^{-0.056} \right] \quad (32)$$

Nomenclature

- $A_c$: Surface area of absorber plate, $m^2$
- $C_p$: Specific heat of air, $J/kg \, K$
- $D$: Equivalent or hydraulic diameter of duct, $m$
- $e$: Roughness height, $m$
- $G$: Mass velocity of air, $kg/s \, m^2$
- $h$: Heat transfer coefficient, $W/m^2 \, K$
- $h_w$: Convective heat transfer coefficient due to wind, $W/m^2 \, K$
- $H$: Depth of duct, $m$
- $I$: Solar insolation, $W/m^2$
- $K$: Thermal conductivity of air, $W/m \, K$
- $K_g$: Thermal conductivity of glass cover, $W/m \, K$
- $K_i$: Thermal conductivity of insulation, $W/m \, K$
- $L$: Length of duct, $m$
- $L_1$: Spacing between covers, $m$
- $L_g$: Thickness of glass cover, $m$
- $M$: Number of glass covers
- $m$: Mass flow rate, $kg/s$
- $\Delta P$: Pressure drop across the duct, Pascal
- $P$: Roughness Pitch, $m$
- $Q_u$: Useful heat gain, $W$
- $T_o$: Outlet air temperature, $K$
- $T_i$: Average of inlet and outlet air temperature, $K$
- $T_c$: Sky temperature, $K$
- $U_L$: Overall heat loss coefficient, $W/m^2 \, K$
- $U_i$: Top loss coefficient, $W/m^2 \, K$
- $U_b$: Bottom loss coefficient, $W/m^2 \, K$
- $U_s$: Side loss coefficient, $W/m^2 \, K$
- $V$: Velocity of air in the duct, $m/s$
- $W$: Width of duct, $m$
- $W_v$: Wind velocity, $m/s$

Dimensionless parameters

- $e/D$: Relative roughness height
- $Re^*$: Roughness Reynolds number
- $Re$: Reynolds number
- $Pr$: Prandtl number
- $f_r$: Friction factor for rough surface
- $F_r$: Collector heat-removal factor
- $F_p$: Collector efficiency factor
- $\alpha/90$: Relative angle of attack
- $N_{a_k}$: Nusselt number for smooth duct
- $N_{a_k}$: Nusselt number for rough duct
- $W/H$: Duct aspect ratio
- $f_s$: Friction factor for smooth duct
- $P/e$: Relative roughness pitch
- $(\alpha a)_c$: Effective transmittance-absorptance product
- $\beta$: Tilt angle of collector surface, degree

Greek symbols

- $\mu$: Dynamic viscosity of air, $Ns/m^2$
- $\alpha$: Angle of attack, degree
- $\rho$: Density of air, $kg/m^3$
- $\sigma$: Stefan-Boltzman’s constant, $W/m^2 \, K^4$
- $\delta_i$: Thickness of insulation, $m$
- $\varepsilon_p$: Emissivity of absorber plate
- $\varepsilon_g$: Emissivity of glass cover
- $\eta_c$: Carnot efficiency
- $\eta_{th}$: Thermal efficiency
- $\eta_{eff}$: Effective efficiency
- $\eta_{ex}$: Exergetic or Exergy efficiency
- $\eta_f$: Fan or Blower Efficiency
- $\eta_m$: Efficiency of Electric Motor
- $\eta_t$: Electrical power transmission efficiency

References:


