QUADRATIC MODULES FOR LIE ALGEBRAS

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Abstract

In this work we give the notion of quadratic module for Lie algebras and explore the connections between this structure, 2-crossed modules and simplicial Lie algebras in terms of hypercrossed complex pairings.

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Introduction

Crossed modules of groups defined by Whitehead in [11] are algebraic models of (connected) homotopy 2-types. The Lie algebra analogue of crossed modules was introduced by Kassel and Loday in [9]. Simplicial groups first studied by Kan, [8], have a well structured homotopy theory and they model all homotopy types of connected spaces. Conduché in [4] defined an algebraic model for connected 3-types. His models, called 2-crossed modules, have very pleasant properties and these 2-crossed modules form a category equivalent to that of simplicial groups with Moore complex of length 2 (cf. [4]). Ellis in [7] captured the algebraic structure of a Moore complex of length 2 in his definition of a 2-crossed module of Lie algebras. This is the Lie algebraic version of a group theoretic notion defined by Conduché.

Within the homotopy theory of simplicial Lie algebras, analogues of Samelson and Whitehead products are given by sums over shuffles \((a;b)\) of Lie products. Akça and Arvasi in [1] explained the relationship of these shuffles to crossed modules and 2-crossed modules of Lie algebras, more precisely, by using the image of the higher order Peiffer elements in the Moore complex of a simplicial Lie algebra, they have constructed a functor from the category of simplicial Lie algebras to that of 2-crossed modules of Lie algebras. Quadratic modules introduced by Baues [3] are algebraic models for homotopy connected 3-types. Baues in [3] constructed a quadratic module from a simplicial group. In this paper we will give the notion of quadratic module for Lie algebras and we give the

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