ON A SECOND ORDER RATIONAL DIFFERENCE EQUATION

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Abstract
In this paper, we investigate the stability of the following difference equation

$$x_{n+1} = \frac{ax_n^4 + bx_n x_{n-1}^4 + c x_n^2 x_{n-1}^2 + dx_n^3 x_{n-1} + ex_n^4}{Ax_n^4 + Bx_n x_{n-1}^4 + C x_n^2 x_{n-1}^2 + Dx_n^3 x_{n-1} + Ex_n^4},$$

where the parameters $a, b, c, d, e, A, B, C, D, E$ are positive real numbers and the initial values $x_0, x_{-1}$ are arbitrary positive numbers.

Keywords: Difference equations, Global stability.


1. Introduction and preliminaries
Consider the following second-order difference equation

(1.1) \[ x_{n+1} = \frac{ax_n^4 + bx_n x_{n-1}^4 + c x_n^2 x_{n-1}^2 + dx_n^3 x_{n-1} + ex_n^4}{Ax_n^4 + Bx_n x_{n-1}^4 + C x_n^2 x_{n-1}^2 + Dx_n^3 x_{n-1} + Ex_n^4}, \quad n = 0, 1, \ldots, \]

where the initial conditions $x_0, x_{-1} \in (0, \infty)$ and the parameters $a, b, c, d, e, A, B, C, D, E \in (0, \infty)$. In this paper we study the global stability of the unique positive equilibrium point, and the boundedness and the convergence of the solutions of Equation (1.1).

Nonlinear difference equations appear naturally, for example, from certain models in ecology, economy, automatic control theory, and they are of great importance in applications where the $(n + 1)^{st}$ state of the model depends on the previous $k$ states.

Recently, there has been a lot of attention given to studying the global behavior of nonlinear difference equations by many authors, See for example [1-3,5-19] and the references cited therein.

Now, we review some definitions (see for example [11-12]), which will be useful in the sequel.

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