AN EXAMINATION OF PROSPECTIVE MATHEMATICS TEACHERS’ REALISTIC APPROACHES WITH DIVISION WITH REMAINDER (DWR) PROBLEMS

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Abstract

The purpose of this study was to investigate prospective mathematics teachers’ realistic approaches during problem solving and posing. In advance of this study, pilot study had been implemented to see its necessity. According to findings of pilot study, the current study was required. The current study is a case study of qualitative research methods. 40 mathematics teacher candidates, who study at a state university in Istanbul, participated in this study. Questions were developed by Chen et al. (2005) were used in the study. Descriptive statistics and percent frequency methods were used in data analyzing. As a result, the study was of high importance for prospective teachers to recognize realistic problems.

Key Words: Problem solving, problem posing, realistic approaches, prospective mathematics teachers, division-with-remainder.

Introduction

The process of education has three main components including teachers, students and curriculum. The extent of the curriculum which is prepared in a way to provide students to think critically and creatively is not important as this curriculum will be carried out by teachers who are the co-actor of the curriculum. Therefore, teacher is the most efficient component in the process of education. Teachers should be able to adopt the innovations into changing curriculum (Arslan, 2006). For example, in 2009 the constructivist approach has been adopted for the curriculum reform. The purpose of this reform is to provide students to think, question and criticize as individuals and to prevent rote learning. For this reason, the topic is not only performing mathematics at the lessons but also it is to associate mathematics with real life.

The problem is defined as a difficulty or an obstacle which must be overcome. In school mathematics lessons, students do not notice whether there is a problem for them while their teachers are motivating them. Students are not interested in solving a problem because of the difficulty of recognizing a problem till their teacher asks them to solve the problem (Jonassen, 2010). According to Jonassen (2010), solving a problem is a cognitive activity.

Wood (1983) emphasized that problem can be solved by using path constraint carefully from initial state. Therefore, appropriate selection process (path) refers to problem solving. Problem solving process consists of two steps that are; understanding the problem and searching for solution of the problem (Newell & Simon, 1972). Ideal problem solver uses four stages to reach solution of problem; understanding problem, determining possible strategies for solving, suitable strategy choosing and looking back. The most common problem solving steps in mathematics education

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were developed by Polya in 1957 (Jonassen, 2010). According to Polya; understanding problem, making a plan for solving, executing the plan and evaluate its effectiveness are needed in problem solving process. Moreover, thinking similar problems, recalling similar problems and heuristic approaches can help to solve the problem.

Problem solving and posing have been searched by many mathematics training researchers. They have argued that problem posing is as indispensable as problem solving in teaching mathematics (Brown & Walter, 2005; Cai, 1998, Stoyanova & Ellerton, 1996)

Solving problems related to real life as well as school mathematics, provide students to find out genuine keys to solutions by keeping them out of memorization (Salman, 2012). Also for problem solving, a person need to use features like genuine ways, creative ideas and top-end thinking skills (Tertemiz & Çakmak, 2004)

Students should be given an opportunity to mingle with non-routine and open ended problems. Students should not pose and solve just only routine problems. They need to be encouraged to pose open ended and non-routine problems (NCTM, 2000). Polya says not using problems except from the routine ones in classroom will lead not to use of imagination and failure of evaluation (Cited by Yazgan & Bintaş, 2005)

Schonfield (1988) stated that solving problems provide students to recognize the processes, operations and alternative solutions. The most commonly used problem in problem solving in which interpretation is occurred is DWR (Division with Remainder). To the DWR question which was asked by National Assessment of Education Progress in 1983 (An army bus holds 36 soldiers. If 1,128 soldiers are being transported to their training site, how many buses are needed?) Only %24 of 13 years old American students used the right solution and made realistic comments (Li &Silver 2000). Then DWR problems have been used for searching students’ realistic approaches in many studies. Silver, Shapiro and Deutsch (1993) asked this bus problem to 200 secondary school students with different numbers. Although more than 60 % of students operated the procedure accurately, because of the students who did not display realistic approach, only 45% of students gave the right answer. In this study, the interpretations other than the requested ones were accepted right (for instance while the result was 13,5 and the answer was 14, a student gave an answer like 13 buses + 1 van, and it was accepted right). In this study the most significant outstanding point indicating the fact that while students were choosing the right operation in DWR problems, they could not give the right answer since they could not interpret the solution.

DWR problems are thought to be as complex and non-routine by many researchers, Verschaffel, Greer and De Corte (2000) stated that real life could be used in both routine and non-routine problems. But in routine problems, problem can be solved by following certain actions, whereas it is not sufficient just to follow certain actions in non-routine problems. The number you get should be interpreted in accordance with real life (Rodriguez, Lago, Hernández, Jiménez, Guerrero & Caballero, 2009). Using real life problems (DWR) provides students to recognize the division with remainder while describing the process of division with remainder to students. In China problems, open ended and related to real life, are used not only for problem solving but also for problem posing. In studies it is presented that students are prone to classical non graphical problems whereas they do not display realistic approach. For example, Verschaffel, De Corte and Lasure (1994) defined two problem types as standard and problematic, and, the students were asked to solve them.

Pete organized a birthday party for his tenth birthday. He invited 8 boyfriends and 4 girlfriends. How many friends did Pete invite for his birthday party? (Standard)
Carl has 5 friends and Georges has 6 friends. Carl and Georges decided to give a party together. They invited all their friends. All of their friends came to party. How many friends were there at the party? (Problematic)

It is observed that students solve the standard problems easily while they have difficulties in problematic problems. DWR problems are often used for searching this realistic approach. Operating the division with remainder accurately is not enough and it needs to be interpreted appropriate for real life. Students have a desire for problem solving with only operations ignoring real life is interpreted as students perceive mathematics as being not related to real life. It is claimed that this situation is originated from the books which are used and the problems were used by teachers (Chen et al., 2010).

Cai (1998) presented the question above to American and Chinese students to solve it. Arıkan and Ünal’s studies about this division with remainder bus problem were presented to 7th grade and 8th grade students. But, only 10, 52% of 7th grade students and 22, 41% of 8th grade students could both choose the right operation and interpreted the result accurately. Related to this subject Inoue (2005) claimed that the problems which students exercise in classroom should be parallel with problems they run into their real lives, so, their approach to mathematics lesson can be affected positively.

In this subject especially in university’s teacher training program, to recognize real life problems and constitute environments in which they can discuss on those problems are crucial (Korkmaz, Gür& Ersoy 2004)

The realistic approach which is displayed by students brings to mind the topic that how mathematics prospective teachers have an attitude towards an approach in DWR problems. Thus the aim of this study is to analyze realistic approach of mathematics prospective teachers by solving and posing DWR problems.

The purpose of this study was to examine prospective mathematics teachers’ realistic approaches during problem solving and problem posing activity. Therefore, it was asked whether mathematics teacher had to display realistic behavior. Because teachers need to know their students’ mathematical understanding and help them to link between mathematics and real life. To help students, teachers are able to pose and solve problems by realistic approaches.

A pilot study was carried on sixth grade students for demonstrating the necessity of the study. 100 sixth grade students participated into the pilot study. A routine problem which was division with remainder problem was asked to the students in standard and non-standard versions.

1. 28 slabs of cake were divided into 5 friends equally. Then how many slabs do friends have?

2. If these friends have 28 balloons-not slabs of cake-, then, how many balloons do friends have?

The required data were collected in written and coded as correct or incorrect. A response was coded as correct when true solution and realistic comment were chosen by students. A response was coded as incorrect when a realistic comment was not made although the operation was implemented properly. After the pilot study, students were asked whether they stated their opinion during the problem solving activity.

We used descriptive statistics for data analyzing of the pilot study. According to the findings, 17 students answered correctly the standard problem while this number is 62 for the non-
standard problem. It was surprising for us since we had assumed exactly the opposite case. 80.34% of the 62 students said that they had often a hazardous opinion in problem solving activity.

The Importance of the Study

Problem solving is known as one of the thinking skills that has to be gained for the individuals (Baykul, 2009; Pressesien, 1985). However, the answer that the students have presented for the first question has given a hint that the problem they have faced in the mathematical lessons has been different from those they have been requiring a realistic approach. The fact that the students find the false answer, which is just resulted from the lack of realistic interpretations, despite they have used the correct operation applications in order to solve the problem itself has also been detected as a finding during the studies carried out in the countries, such as Belgium, China, Switzerland, UK and Japan (Greer, 1993; Hegarty, Mayer & Monk 1995; Verschaffel, De Corte & Lasure 1994; Yoshida & Verschaffel and De Corte, 1997; Xin, Lin, Zhang & Yan, 2007).

The fact that the problems requiring realistic approaches have not been used in the lessons or they have very rarely been used has been shown as the reason for this finding. The problems presented by the teachers in the lessons have been strengthening the students' beliefs, such as "each problem has one single solution.", "problems are solved with the given figures.", etc. (Xin et al., 2007). Therefore, it has been important that the teachers have to select the problems from various types for their students. The approaches that the prospective teachers in the teacher training programs presented during the problem posing and solving have been a puzzle just at this point.

Method/Procedure

In this study, case study method among qualitative research approaches was used. Case study method is a kind of systematic design that includes steps like data collection on a certain subject, organizing collected data, interpreting and accessing research findings (Merriam 1998). Although organizing collected data is difficult, it is possible to search the current situation blow by blow and get tangible evidences (Cohen & Manion, 1994).

Participants

40 mathematics prospective teachers, who study at a public university in Istanbul, attended to this study. Prospective teachers stated that they had not carried out studies on problem posing before. Convenience sample was used for selecting participants. The participants were easy to recruit for the study and the researcher did not consider selecting participants that are representative of the entire population.

Insurement

In this study, questions is quoted from the study of Chen, Van Dooren, Chen and Verschaffel (2010). Problem solving and problem posing tasks consist 3 items was conducted with each prospective teachers. First two problems at problem solving task are problematic problems and the last problem is a standard one. At problem posing task prospective teachers were asked to pose a problem that requires 100/8 operation and given numbers as result. Problem solving and problem posing tasks are as in the following.
Table 1. Problem solving and problem posing tasks

<table>
<thead>
<tr>
<th>Problem Solving Task</th>
<th>Problem Posing Task</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 100 children are being transported by minibuses to a summer camp at the seaside. Each minibus can hold a maximum of 8 children. How many minibuses are needed?</td>
<td>Pose a problem of which solution is $100 \div 8$ operation and answers are those numbers below:</td>
</tr>
<tr>
<td>2. Packages of milk are brought to a school. To carry one case, 8 children are needed. How many cases can be carried by the 100 children at one time, if they all cooperate?</td>
<td>13</td>
</tr>
<tr>
<td>3. In a school restaurant 100 liters soup are boiled in equal 8 liters capacity pots. How many liters do these pots contain?</td>
<td>12</td>
</tr>
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</table>

A semi-structured interview was conducted with 5 prospective teachers chosen, in parallel with the results derived from the study.

**Data Coding**

Answers in problem solving were coded as right or wrong. Descriptive statistics and percent frequency methods were used. According to the content analysis, posed problems have classified as (a) realistic, (b) realistic but ill-defined, (c) non-realistic problems. At posing problem, outstanding prospective teachers’ perception for the necessity of problems requires realistic approach and their thoughts about the problem they posed were examined by having a semi-structured interview. Descriptive analysis technique was used in interview analyzing. Interview questions were envisaged by prospective teachers’ responses. Hence, prospective teachers could take the opportunity to see their mistakes.

**Findings**

Out of 40 mathematics prospective teachers who attended to this study, 2 of them could not answer at least one question at problem solving task and 18 of them could not answer questions at problem posing task accurately. Prospective teachers who had difficulties in problem solving and posing were displayed through coding in Table 2 and Table 3.

While two of the prospective teachers could not display realistic approach in problem solving, this number was defined as 5 in problem posing.

Table 2. Unsuccessful students at solving division problems

<table>
<thead>
<tr>
<th>Problem Solving 1</th>
<th>Problem Solving 2</th>
<th>Problem Solving 3</th>
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<tbody>
<tr>
<td>S14, S17</td>
<td>S14, S17</td>
<td>S14</td>
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</table>

Table 3. Unsuccessful students at posing division problems

<table>
<thead>
<tr>
<th>Problem Posing</th>
<th>Non Realistic</th>
<th>Realistic but ill-defined</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem Posing 1</td>
<td>S12</td>
<td>S2, S4, S5, S6, S9, S11, S14, S17</td>
</tr>
<tr>
<td>Problem Posing 2</td>
<td>S12, S15, S16</td>
<td>S1, S2, S4, S8, S9, S10, S14, S17</td>
</tr>
<tr>
<td>Problem Posing 3</td>
<td>S7, S18</td>
<td>S3, S10, S13, S14</td>
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</table>
When we analyze Table 2 and Table 3 together, we see teacher candidates who cannot solve and pose the same type of questions. It is possible to comprehend that prospective teachers have not done any study about problem posing before. 45 % of the prospective teachers failed.

Five teacher candidates, S7, S12, S15, S16, S18 posed problems which were not related to real life when they posed problems. It was detected, although 18 prospective teachers who had not been successful on problem (DWR problem) posing study, tried to pose problems with realistic approach, they had linguistic deficiencies.

For instance;

S11: A group of 100 friends is going to ride on a swing for 8 people, to have fun. According to this, how many times does it have to run for riding each person?

1 toy= 8 friends
2 toys= 2x8 friends
3 toys= 3x8 friends
.
.
.
12 toys=12x8 friends
13 toys= 97,98,99,100 So it has to run 13 times.

While the prospective teachers using the word ‘swings’ in problem, explained it as groups of 8 friends ride on the swing in turn. This situation gives the impression that prospective teacher’s thoughts and writings are not consisted with each other, in other words he cannot put his thoughts into his words.

S2: Ahmet needs 8 people to donate blood, for taking blood which is compatible to his blood type. How many tubes of blood can he take if he brings 100 people?

In this question prospective teachers seem to think if 8 people can give one tube of one blood type, 100 people can give 12 tubes of his blood type. But the problem does not represent that aim. A student can give answer to this question as 100 tubes.

S13: A woman prepares 100 liter fruit juice and will share it with her 8 neighbors. How many liters does it fall to each one’s share?

In this question prospective teacher does not express how many liter that will be shared to whom or if it will be equal. So, many possibilities come into question for the solution.

Samples of unstructured problems as follows:

S4: There are desks for 8 in a 100 seat classroom. When all the desks are full, there are students remain standing. How many desks are there in classroom?

If the classroom is for 100 students, how some students can remain standing?

S10: Maximum 8 students can attend to each school club. According to this, how many students cannot attend to no club? How many students remain without a club?
If teacher candidate had asked this question as ‘how many school clubs are there?’”, he would have posed the problem appropriate for the requested situation.

Samples of problem which did not display realistic approach:

S12: A teacher handed a 100 lined poem in his students and ask them to write it on their notebooks. Students’ notebooks are same and there are 8 lines in a page. How many pages are required to write this poem? (Problem Posing 1)

Prospective teacher may think that the pages could be written to halfway. Thus the answer could be 13. But the page could be indicated ‘half’. So the result could be as 12 whole and a half page. So the answer would be 12, 5.

S12: There are 8 tables in a cafeteria. Students will have lunch there at lunch time. They will sit down equally around each table in this school for 100. How many students can sit down around each table? (Problem Posing 2)

Teacher candidate may think that 4 students do not have lunch. Because students cannot be divided into two parts.

S7: We have 100 liters of water. There are 8 people who can drink water at same capacity. 100 liters water is shared to those 8 people. How much water can each person drink? (Problem Posing 3)

Water drinking capacity cannot be 12,5 liters.

S18: Some pools will be opened in a sport complex. 8 isochoric pool takes 100 liters water. How many liters do the pools take?

Semi-Structured Interviews:

Structured interview carried out with 5 teacher candidate among the teacher candidates who give conspicuous answers. The researcher who carried out the interview was coded as R. Structured interview carried out with the prospective teachers who cannot pose a problem.

Problem which was posed by S10 for Problem Posing2

Maximum 8 students can attend to each school club. According to this, how many students cannot attend to any of the clubs? How many students will be without a club?

R: Do you think the problem you posed is appropriate for the requested situation?
S10: It coincided with the end of lesson. I wrote it wrong because of hurry.
R: Why wrong?
S10: I think the main question would be ‘ how many clubs are there in school?’
R: Do you think problem solving and posing study makes a professional contribution to you?
S10: One of our teachers showed us a video about Chinese education system. Students dwelled on only one question at one lesson; they found out different solutions and posed different problems similar to that one. After that video I came to believe problem posing is a more effective method. In our education system different problems confront to our students but the solutions are always the same, after some time the students begin to learn those solutions by heart.

Answer of S17 for Problem Solving 1
There are 100 students. If each minibus takes 8 people, one of them must be the driver. So, each minibus takes 7 students. Then the solution is 100/7 = 14. Thus they need 15 minibuses.

R: Why did you take the driver into consideration in problem solving?
S17: I wanted a different solution. I am sure others in classroom solved the problem as you wanted.

R: Do you think problem solving and posing study makes a professional contribution to you?
S17: Honestly, I do not think that will contribute me.

Problem which was solved by S7:
We have 100 liters of water. There are 8 people who can drink water at the same capacity. 100 liters water is shared to those 8 people. How much water can each person drink?

R: Can you help me to solve the problem you posed?
S7: Yes, I think I posed a nice problem, answer is 12.5 liters. (S7 laughs)
R: All right. Can we have a capacity of drinking 12.5 liters of water?
S7: Of course we cannot.
R: Do you think we should reflect the reality in problems we pose? Or the problem posed after the operation and result is sufficient?
S7: I think it should, for instance, when we learn a subject at first, we search the samples in real life. It is like that.
R: Finally, do you think problem solving and posing study makes a professional contribution to you?
S7: Of course it does. For example, here after, I will search if it is sensible or not when I pose a problem (S7 laughs again).

Problem which was posed by S11 for Problem Posing1:
A group of 100 friends is going to ride on a swing for 8 people, to have fun. According to this, how many times does it have to run for riding each person?

R: I was confused while solving this problem. Are you sure you posed it accurately?
S11: I am sure it is true and an eliminative one.
R: While there was the term ‘swings’ in problem, I understood as if there were one swing and all the students rode on the swing in a group of 8 at solution. Can you explain the solution clearly?
R: Do you think problem solving and posing study makes a professional contribution to you?
S11: Sure I think problem posing, solving, get students to pose and solve problem are important for mathematics teachers. Because we both learn how to react to the cases related with the real life, and improve our minds using our minds. As a prospective teacher; from our private lesson experiences at least I observed students have difficulty in problem solving. First reason of it, we do not teach them how to pose a problem, we give them the problem on hand and ask them to solve it. Second reason is our teachers who were educated with traditional method reflect us as if questions have just one solution. We, as newly educated prospective teachers, should improve ourselves with this kind of studies at first, and then we should apply this training we got to our students.
Problem of S1 for Problem Posing 3

We have 100 balls. There are 8 boxes to put them in. On condition that we put equal numbers of balls in these boxes; how many balls are there in a box?

R: Is this problem appropriate for the requested situation?
S1: No, it is not actually. But if I had more time, I could pose a creative problem.

R: Why is this problem not appropriate?
S1: 100 liters water would be more appropriate. Balls cannot be divided.

R: Do you think problem solving and posing study makes a professional contribution to you?
S1: It will not, if I pose the problem inaccurately.

Problem of S18 for Problem Posing 3

Some pools will be opened in a sport complex. 8 isochoric pools take 100 liters water. How many liters do the pools take?

R: I could not solve this problem.

S18: What kind of help do you want exactly? Should I help about how the students will solve it? Then I draw 8 isochoric pool on the board. Since they are at the same volume I code them x. Totally, there are 100 liters I write an equation as x+x+x+x+x+x+x=100. Then, I try to explain it to students.

R: But how can we swim in 12, 5 liters water?
S18: Oooww yes! Very sensible. Then the number has to be increased or a problem related to milk can be posed.

R: Have you ever thought it should be meaningful with real life when you pose a problem?
S18: Yes it exactly should be. We can say carelessness, inexperience at problem posing.

R: Do you think problem solving and posing study makes a contribution to you vocationally?
S18: Problem solving and posing are important. Problem should be posed by regarding level of students. Age, learning level and intelligence capacity of students determine that level. The more students solve problem the better they comprehend the subject, problem solving improves the mind. I think for this reason problem solving and posing at lessons are important.

Conclusion

According to pilot study, sixth graders thought that cake was divisible. Therefore, they approached realistically during problem solving. In other words, the result of pilot study supported the necessity of the study.

It is mentioned by researchers that there are deficiencies at teacher training and they need to be removed by analyzing them well. One of the components of teacher’s learning is stated as knowledge of students' development of particular mathematical ideas (NCTM, 1991). Training of prospective teachers, at a level of reaching professional competence in pre-service education is among the aims of teacher training program (İşık, Çıtaş & Baş, 2010) Providing qualified education at schools can be possible with the teachers who are contained in school (Seferoğlu, 2004). For this reason, continuation of training without being aware of the existing curriculum can cause problems when prospective teachers enter the profession. Problem solving and posing have
importance in mathematics curriculum of which first step was taken in 2005 and of which reform was accelerated in 2009. Therefore, for teacher candidates, experiencing the applications of problem solving and posing as lessons help to overcome the problems.

Referring to Table 2 and Table 3, although prospective teachers are 95% successful at solving problem which required division with remainder, their success at posing those problems is defined % 45. Therefore, teacher candidates can solve non-routine problems using their experiences they have gained up to the present. However due to being inexperienced or lack of assessment by an expert, they can be said to be far below the level required in problem posing. If the teacher candidates do not confront with the deficiencies and do not correct those deficiencies, their students can avoid problem posing or teacher himself may become dependent on textbooks and reference books instead of problem posing.

One of the outstanding point in study is the answer to interview questions which S17 gave. Teacher candidates’ perceptions stands out in their works. Teacher candidate stated that he wished to offer a different solution and wanted to give an answer out of cliché. Of course, it is important to have individuals who think critically, produce creative solutions and question. However, what these concepts are, should be explained to the teacher candidates with classroom interactions.

Another outstanding point in this study is posing problems of teacher candidates which is not related to real life. At the interview they realized that they posed problems not related to real life when they posed problems. This situation can be caused by teacher candidates who approach mathematics as operational knowledge. Teacher candidates accepted priority in aims, is emergence of a well-structured problem and they ignore posing a problem that can be solved by knowing which operation should be made for which reason. Concept knowledge is to know the meaning of operations and generalization made (Bekdemir & Işık 2007). Therefore no matter how accurate the result is operationally, capacity of drinking 12.5 liters water and swimming in 12.5 liters water in a pool cause students to perceive mathematics as an independent concept of reality. Problems completely contrary to the purpose of constructivist education and reflect mathematics as a tangle of operations should be avoided.

A similar study was conducted by Chen et al (2011) and the researchers found that the participants had behaved realistically when solving and posing DWR problems. Cooper and Harries (2003) conducted a study on DWR problem for solving and interpreting. The participants of their study was 14 working class students who were 11-12 years old. The participants were presented DWR problem at first step and s they responded four yes-no questions which included the DWR problem to make explicit of students’ interpretations. Furthermore, The result of this study, when the students faced with DWR problems, they could not identify proper operations.

Based on the survey, as long as mathematics teachers approach realistically, it will be satisfied with expected mathematical performance for secondary students.
References


