AN OVERVIEW ON PULSATILE FLOW DYNAMICS

*Melda Özdinç Çarpinlioğlu
University of Gaziantep, Faculty of Engineering, Department of Mechanical Engineering
27310, Gaziantep, TURKEY

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*M.Ö. Çarpinlioğlu, Tel:+90-342-317-2509, Fax: +90-342-360-1104
E-mail address: melda@gantep.edu.tr

ABSTRACT
Pulsatile flow dynamics in reference to the relevant experimental research on the manner between the time periods of 1997-2015 is presented in this paper. The flow field under discussion is generated through a rigid circular cross-sectional pipe as an axial slightly- compressible and sinusoidal one in a controlled range of the oscillation parameters. Laminar and turbulent flow regimes are considered with a particular emphasis devoted to the transitional characteristics of laminar pulsatile flow into turbulence. The definitions of the flow characteristics, the methodology of the experimental studies and the results on the physical nature of the field are described. The departure of pulsatile flow from a steady one is presented in reference to the velocity field and frictional field analyse under the influence of oscillation frequency and amplitude. In this context, the critical magnitudes of Womersley number, \( \sqrt{\frac{A}{\rho f^2}} \) which is a non-dimensional frequency parameter are such that below \( \sqrt{\frac{A}{\rho f^2}} = 1.32 \) quasi-steady region and above \( \sqrt{\frac{A}{\rho f^2}} = 28 \) an inertia dominant regions are well-defined. The so-called intermediate region is inside the range of \( 1.3 < \sqrt{\frac{A}{\rho f^2}} < 28 \). However, the influence of oscillation amplitude, \( A_1 \) is not discussed up to the time of cited research of our team [4, 5]. Inside the quasi-steady region, steady flow fundamentals can be used with an acceptable accuracy since the physical flow nature in laminar pulsatile flow and turbulent pulsatile flow resemble the characteristics of steady flow. However transitional characteristics of pulsatile laminar pipe flows still have a lot of unknown facts as was discussed in [6]. The experimental studies are based on the visual observations of the local velocity waveforms, the friction factor variation and the detection of the disturbance growth [7]. It is also known that the turbulence generation mechanism is different from that of a steady flow case. The interactive influences of \( f \) and \( A_1 \) in laminar , turbulent regimes and through the passage from laminar flow into turbulence are described in reference to the controlled flow field...
measurements conducted previously in order to have an overall analysis of the pulsatile flow fields.

Nomenclature of pulsatile flow

A sinusoidal pulsatile flow is defined as a periodic flow which is generated by the superposition of an oscillation over a steady flow. The sub classes of periodic flow are the so-called oscillating and pulsatile ones as can be seen from Table 1 of [7]. The flow physics is governed by the following momentum-integral equation which is known to be valid irrespective of the flow regime:

\[
\rho \frac{d\tilde{u}_m(t)}{dt} + \frac{4\tau_{in}(t)}{D} = \frac{\Delta P(t)}{L}
\]  

(1)

The direct measurements of \(\tilde{U}_m\) and \(\Delta \tilde{P}/L\) are approximated by the following finite Fourier expansions:

\[
\tilde{U}_w = \tilde{U}_{m,0} + \sum_{n=1}^{N} \tilde{U}_{m,n}\cos(\omega t + \angle \tilde{U}_{m,n})
\]  

(2.a)

\[
\Delta \tilde{P}/L = \Delta \tilde{P}_{,0} + \sum_{n=1}^{N} \Delta \tilde{P}_{,n}\cos(\omega t + \angle \Delta \tilde{P}_{,n})
\]  

(2.b)

The usual practice which does not have an accuracy loss is to use an approximation by the first harmonics of the wave as follows:

\[
\tilde{U}_w = \tilde{U}_{m,0} + |\tilde{U}_{m,1}|\cos(\omega t + \angle \tilde{U}_{m,1})
\]  

(3.a)

\[
\Delta \tilde{P}/L = \Delta \tilde{P}_{,0} + |\Delta \tilde{P}_{,1}|\cos(\omega t + \angle \Delta \tilde{P}_{,1})
\]  

(3.b)

In these definitions \(\tilde{U}_{m,0}\) is the cross-sectional mean velocity, \(\tilde{U}_{m,1}\) is the time-averaged value of cross-sectional mean velocity, \(\tilde{U}_{m,1}\) is the amplitude of cross-sectional mean of oscillating velocity, |\(\Delta \tilde{P}_{,0}\)| is the instantaneous pressure drop with the corresponding time -averaged and oscillating components of |\(\Delta \tilde{P}_{,0}\)| and |\(\Delta \tilde{P}_{,1}\)|, \(t\) is the time coordinate, \(\omega\) is the phase angle, \(\omega\) is the angular frequency of oscillation, \(\omega = 2\pi f\) and \(f\) is the oscillation frequency.

It is well known that the practical flow nature determination is in reference to the magnitude of flow Reynolds number. The referred steady pipe flow Re number, \(Re = UD/v\), is based upon characteristic steady velocity, \(U = \tilde{U}_m\) where \(D\) is the pipe diameter and \(v\) is the kinematic viscosity. In pulsatile flow terminology the accepted definitions of time-averaged Reynolds number, \(Re_{ta}\) and oscillating Reynolds number, \(Re_{os}\) are as follows:

\[
Re_{ta} = \frac{|\tilde{U}_{m,ta}| D}{v}  
\]  

(4.a)

\[
Re_{os} = \frac{|\tilde{U}_{m,os}| D}{v}  
\]  

(4.b)

Therefore time-averaged Reynolds number, \(Re_{ta}\) can be treated as the counterpart of Re. Meanwhile the non-dimensional parameters expressing the oscillation characteristics are given with Womersley number, \(\sqrt{\omega}/v\) and oscillation amplitude, \(A_1\) defined as follows:

\[
\sqrt{\omega} = R\frac{\sqrt{\omega}}{v} 
\]  

(5.a)

\[
A_1 = \frac{|\tilde{U}_{m,os}|}{\tilde{U}_{m,ta}}  
\]  

(5.b)

where \(R\) is the pipe radius.

The experimental research on pulsatile flow necessitates the direct measurements of instantaneous velocity and instantaneous pressure which make the calculation of instantaneous shear stress, \(\tau_{in}(t)\) in turn. The measurements are also evaluated to calculate instantaneous friction factors. Therefore pulsatile flow field analysis is through \(Re_{ta}\), \(Re_{os}\) which are under the influence of \(\sqrt{\omega}\) and \(A_1\) instead of the single reference parameter of Re in a steady flow field.

Outline of the experimental research

The design and construction of the separate experimental test systems are presented in details [4, 5, 8, 9, and 10]. However the comparison of the test systems and the generated pulsatile flow fields should be taken into account to start with. The sketches of the test systems can be found in Figure 1. The test systems are open circuit ones through which air flow is generated by suction type [4] and blowing type arrangements [5]. The rigid and smooth PVC pipes of \(D=50.4\) mm and \(D=26.6\) mm are used in each system. The primary difference of the test systems come from the method of oscillation generation. In the first system an oscillation generator in the form of a reciprocating piston driven by a scotch yoke mechanism is used. In the second test system an electronic mass flow controller unit, MFC coupled to a screw air compressor generates the sinusoidal air flow. The error in the generation of sinusoidal air flow is about 3% and 1.2% in the first and second system respectively. The test systems generate an extensive range of pulsatile air field in the controlled magnitudes of oscillation frequency and amplitude. The measurements of local instantaneous velocity wave forms and local instantaneous pressures are used at the test station which is far from the flow development effects. Instantaneous velocity measurements are conducted inside the pipe cross section using DANTEC –CTA system. The pressure transducers of inductive difference type HBM-PD1 [4] and local static pressure ones of type WIKA-SL1 [5] are used. The velocity and pressure measurement sensitivity is ± 0.15% and ± 1% respectively. The data acquisition systems have I/O board KEITHLEY, DAS 1602[4] and IO TECH Daq 3001 USB board [5] associated with PC. Table 2 outlines the comparison between the test systems.
and data acquisition and basic measurement sensitivities. The local instantaneous velocity wave fronts and instantaneous local pressure measurements are referred to verify the pulsatile flow nature. Therefore generation of the flow with the specified frequency, amplitude and the form is of primary importance. The instantaneous velocity wave fronts are expressed in terms of cross sectional instantaneous velocity profiles which can also be given as oscillating and time average components separately.

The instantaneous velocity measurements enable us to calculate the cross sectional time average and oscillating velocity components which are used in the calculation of Re_{os} and Re_{ta} in turn. Since steady flow basics are the comparison base for pulsatile flow regime laminar flow resembling Blasius profile and turbulent flow resembling Prandtl’s 1/7 th power law are looked at for the verification of pulsatile flow regime. The instantaneous pressure measurements make the calculation of instantaneous local pressure loss. In the first investigation a local pressure difference is measured at the velocity measurement station while in the second one pressure measurement stations including the velocity measurement station are used. The instantaneous shear stress and the instantaneous flow friction factors can be calculated in reference to velocity and pressure measurements by using the Eq.1 and the definitions of friction factors given in nomenclature.

In the first investigation [4, 8] the range of Re_{ta} is selected between 2000-60000 to determine the pulsatile flow characteristics in laminar and turbulent ranges. Meanwhile in the second [5, 9] investigation the primary research is focused on transitional characteristics of pulsatile flow field under the influence of oscillation parameters of frequency and amplitude at the departure from laminar flow regime. Therefore the corresponding range of Re_{ta} is selected between 1019- 4817.

<table>
<thead>
<tr>
<th>Type of flow</th>
<th>Steady</th>
<th>Oscillating</th>
<th>Periodic</th>
<th>Pulsatile</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A typical velocity-time record</strong></td>
<td><img src="image1.png" alt="Graph" /></td>
<td><img src="image2.png" alt="Graph" /></td>
<td><img src="image3.png" alt="Graph" /></td>
<td><img src="image4.png" alt="Graph" /></td>
</tr>
<tr>
<td><strong>Velocity</strong></td>
<td>$U$</td>
<td>$U_{m,os} = 0$</td>
<td>$U_{m,os} = 0$; $U_{m,ta} = 0$</td>
<td>$U_{m,os} = 0$; $U_{m,ta} = 0$</td>
</tr>
<tr>
<td><strong>Frequency</strong></td>
<td>$f = 0$</td>
<td>$f \neq 0$</td>
<td>$f \neq 0$</td>
<td>$f \neq 0$</td>
</tr>
<tr>
<td><strong>Womersley number</strong></td>
<td>$\sqrt{\omega'} = R\sqrt{\omega/v} \neq 0$; $\omega = 2\pi f$</td>
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<td>$\sqrt{\omega'} = R\sqrt{\omega/v} \neq 0$; $\omega = 2\pi f$</td>
<td></td>
</tr>
<tr>
<td><strong>Amplitude</strong></td>
<td>$A_t = 0$</td>
<td>$A_t = \frac{U_{m,os}}{U_{m,ta}}$</td>
<td>$A_t = \frac{U_{m,os}}{U_{m,ta}}$</td>
<td>$A_t = \frac{U_{m,os}}{U_{m,ta}}$</td>
</tr>
<tr>
<td><strong>Reynolds number</strong></td>
<td>$Re = \frac{UD}{v}$</td>
<td>$Re_{os} = \frac{U_{m,os}D}{v}$; $Re_{ta} = 0$</td>
<td>$Re_{os} = \frac{U_{m,os}D}{v}$; $Re_{ta} = \frac{U_{m,ta}D}{v}$</td>
<td>$Re_{os} = \frac{U_{m,os}D}{v}$; $Re_{ta} = \frac{U_{m,ta}D}{v}$</td>
</tr>
</tbody>
</table>
Figure 1.A. Sketch of the blowing type test set up used for MF09-09

Figure 1.B. Sketch of the suction type test set up used for MF97-04
Figure 2A. Instantaneous velocity wave front at $Re_t=3018$, $\sqrt{\omega'}=2.72$, $A_1=0.53$ as a verification of FFT for measured $Um$ [14]

Figure 2B. Velocity wave fronts at $f=1$Hz in the range of $Re_u=2233\pm76$, $Re_{\omega}=754\pm26$, $\sqrt{\omega'}=8.61$ and $A_1=0.34\pm0.012$ taken at pipe centerline $r=0$ and at pipe wall, $r/R=0.977$[14]

Figure 3. Pressure wave fronts for a measurement period at $f=1$ Hz in the range of $Re_u=2233\pm76$, $Re_{\omega}=754\pm26$, $\sqrt{\omega'}=8.61$ and $A_1=0.34\pm0.012$ taken at 7 axial positions along the flow direction [14]

Figure 4. Cross sectional profiles, $U_a$ and $U_o$ at $f=1$ Hz in the range of $Re_u=2233\pm76$, $Re_{\omega}=754\pm26$, $\sqrt{\omega'}=8.61$ and $A_1=0.34\pm0.012$ [14]

The sample measurements on local velocity wave fronts, local pressure waves along the flow direction and the test section cross-sectional velocity distribution based upon the data collected [5,9] are given in Figures 2 to 4. The data acquisition based upon the measurements is through the execution of an interactive program prepared in Quick Basic language in the first investigation. Meanwhile in the second investigation a program prepared in Labview 2009SP environment is used. [10, 11, 12, 13, 14, 15]

Figure 5A. Sample plots for wall shear stress calculation $Re_{\omega} = 2416$, $Re_{\omega} = 438$, $\sqrt{\omega'} = 2.72$ and $A_1 = 0.18$[9]
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Figure 5B. Sample laminar velocity wave fronts for $Re_\text{lu}=2347$, $Re_\text{lu}=2248$, $\sqrt{\omega'}=2.72$, $A_1=0.96$ at $(r/R=0.977)$ and $(r/R=0)$ [9]

Figure 5C. Sample cross sectional non-dimensional velocity profiles [9].

<table>
<thead>
<tr>
<th>Research Project</th>
<th>Oscillation Generator</th>
<th>Diameter (D mm)</th>
<th>Velocity Pressure Measurement (Sensitivity)</th>
<th>Data Acquisition System (Data Accumulation)</th>
<th>$\sqrt{\omega'}$</th>
<th>$A_1$</th>
<th>Calculated Sensitivity in $\sqrt{\omega'}$</th>
<th>Calculated Sensitivity in $Re_{\text{lu}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MF 97-04[4]</td>
<td>SketchYoke Mechanism</td>
<td>50.4</td>
<td>DANTEC CTA 56C01 ($\pm 0.15%$) Difference, HBM-PD1 ($\pm 1%$)</td>
<td>I/O board KEITHLEY,DAS 1602, 100 kilosamples/s (30 phases of oscillation, 200 cycles at each phase, 6000 data readings)</td>
<td>$7\pm 28.0$</td>
<td>$0.05-0.8$</td>
<td>$\pm 0.01%-4%$</td>
<td>$\pm 1.5%-17%$</td>
</tr>
<tr>
<td>MF 09-09[5]</td>
<td>Electronic MFC Unit</td>
<td>26.6</td>
<td>DANTEC CTA 56C01 ($\pm 0.15%$) Local, WIKA-SL1 ($\pm 0.8%$)</td>
<td>IO TECH Daq 3001 USB 100 Hz (5000 data readings)</td>
<td>$2.72-32.21$</td>
<td>$0.05-0.96$</td>
<td>$\pm 1.2%$</td>
<td>$\pm 3.4%$</td>
</tr>
</tbody>
</table>
Pulsatile Flow Nature

In the first investigation, turbulence level of the flow was measured in the range of 0.8% -1.5% for the covered Re range in the pipe cross section with a minimum value at centerline and a maximum one close to the surface. Steady flow has the critical Re numbers as 5300 and 19000 for the start and end of transition based on the measured cross sectional velocity distributions approximated by Blasius and Prandtl 1/7th power laws for laminar and turbulent flow nature respectively. It is found that steady flow is fully laminar for Re<5300 and fully turbulent for Re>19000. Therefore transitional range is determined to be 5300< Re <19000. The generation of sinusoidal pulsatile flow was through the test cases of 90 different combinations of Womersley number and oscillation amplitude. The measurements of cross-sectional velocity profiles inside sinusoidal pulsatile flow in the range of 7≤√/g<28 and 0.05≤A<0.8 resulted in the respective magnitudes of Re<crit; 17929 and 23763 as the critical limits of Re>crit for the start and end of transition. The addition of oscillations is coupled with a significant delay in transition by extending the laminar flow region. It is also observed that for Womersley numbers of 7 and 12.5 a flow reversal was observed with A<1.0. The estimation of the flow reversal is such that the relationship between Re< and A implies a critical magnitude of A= 0.2. Since for A< less than 0.2 the range of Re< is between a great band from laminar to turbulent regimes (from 2000 to 60000) while for A=0.2, Re< decreases to 2000 with purely laminar regime existence. The critical magnitude of Re< for the cases tending towards flow reversal is found as 738 roughly.

In the second investigation, turbulence level of the flow was measured in the range of 0.5% -1.5% for the covered Re range. The generation of sinusoidal pulsatile flow was through the test cases of 227 different combinations of Womersley number and oscillation amplitude such that 199 test cases were at the onset of transition start as a departure from laminar regime.Fully laminar pulsatile flow was studied as 28 test cases. Steady flow has a critical Re as 2450 at the start of transition. The steady flow is fully laminar for Re<2450. The utilization of sinusoidal pulsatile flow in the range of 2.72≤√/g<32.21 and 0.05≤A<0.96 resulted in the respective magnitudes of Re<crit; ranging between 2700 and 4817 at the start of transition. Therefore depending on the magnitude of Womersley number and amplitude of oscillation laminar flow region is extended over the one of steady flow.

Influence of oscillation parameters

There exists an interactive influence of the oscillation parameters. The covered oscillation frequencies are between 0.1 Hz and 3 Hz in the first investigation and 0.1 Hz and 14 Hz in the second investigation. The change in the diameter of the test pipe resulted in the covered band of the Womersley numbers. Therefore non-dimensional parameter of Womersley number describes the non-dimensional frequency as a governing term since the distinction between low and high magnitudes of frequency should be better expressed by Womersley number.

Flow field dynamics is sensed in terms of the following basic measurement means (Figure 5):

1) Local velocity wave fronts taken as instantaneous measures at pipe centerline and close to the pipe wall at r/R=0.977.
2) Cross sectional mean velocity expressed as U<sub>m</sub> by evaluation of local velocity wave fronts
3) Cross sectional velocity profiles expressed as relevant terms for the time-average and oscillating components to describe the deviation from well-accepted steady counterparts
4) Local pressure wave fronts taken as instantaneous measures along the flow direction which can be expressed as instantaneous local shear stress calculations
5) In reference to the measurements and calculations on velocity, pressure and shear stress data local frictional field measures in terms of defined friction factors.

In reference to the cited experimental research, with the oscillation frequency band as 0.1 Hz< f <14 Hz in the range of the oscillation amplitudes as 0.05<A<0.96 the following conclusions can be given:

1) The influence of oscillation parameters on the abovementioned measurement means have shown a drastic physical change in flow dynamics. This flow behavior change is not dependent on the flow regime. The critical magnitude of oscillation frequency causing this behavior change is sensed by non-dimensional frequency parameter of Womersley number√/g. The critical magnitude of Womersley number seems to be √/g = 8.61.
2) Due to the governing influence of the flow field geometrical plane –diameter of the pipe on the generated pulsatile field, low frequency and high frequency distinction should be better with the magnitudes of √/g.
3) The regional specification of pulsatile flow is such that I) quasi-steady one for √/g< 2.72
II) Intermediate one for 2.72≤√/g< 27.22
III) inertia-dominant one for √/g > 27.22
4) Similar to the low-high frequency range low and high amplitudes of oscillation can be described as follows:
I) A<1 is the high magnitude of oscillation producing flow reversal which could not be measured
II) The functional relationships between a variety of measurement means are such that A<0.3 can be described as the low magnitude of oscillation amplitude
5) Flow regime change from laminar to turbulent flow is such that the first turbulent bursts are observed in the decelerating phase of oscillation by visual treatment and numerical confirmation on the local velocity wave fronts.
6) The location inside the pipe is also having a severe influence on turbulence generation.
7) The utilization of oscillations to control laminar flow seems to be possible.

Conclusions
The basics of flow dynamics of a pulsatile flow field through a pipe are described experimentally and confirmed by available theoretical background. The pulsatile flow can be used as a control mean for the extended laminar regime it offers from hydrodynamics point of view. The critical magnitudes of frequency and amplitude of oscillation which change the physical nature of the flow field are described. However Womersley number seems to have a dominant influence. The geometrical plane of flow in terms of pipe diameter seems to have a significant influence on oscillations.

Eventually the influence of oscillations in heat transfer and thermal development inside channel flows seem to be a candidate research topic.

NOMENCLATURE

\[ A_1 \] velocity amplitude ratio, \[ A_1 = \left| U_{m, os, 1} \right| / U_{m, ta} \]

\[ D \] pipe inner dia meter, m

\[ f \] frequency of oscillation, 1/s, Hz

\[ I \] turbulence intensity,

\[ L \] axial distance between pressure transmitters, m

\[ N \] number of periods

\[ P(t) \] instantaneous local static pressure, Pa

\[ \bar{P} \] Ensemble-averaged static pressure, \[ \bar{P} = \frac{1}{1000} \sum P_i \], Pa

\[ \bar{P}_{os, 1} \] Oscillating component of pressure, Pa

\[ P_{ta} \] Time-averaged component of pressure, Pa

\[ r \] radial position from centerline, m

\[ R \] pipe radius, D/2 m

\[ \text{Re} \] steady flow Reynolds number, \[ \text{Re} = UD / \nu \]

\[ \text{Re}_{os} \] oscillating Reynolds number, \[ \text{Re}_{os} = \left| U_{m, os, 1} \right| D / \nu \]

\[ \text{Re}_{ta} \] time-averaged Reynolds number, \[ \text{Re}_{ta} = U_{m, ta} D / \nu \]

\[ t \] time coordinate, s

\[ \bar{u}_{os} \] Oscillating component of local velocity in x-axis, m/s

\[ \bar{u}_{ta} \] Time-averaged component of local velocity in x-axis, m/s

\[ U(r, t) \] Instantaneous axial velocity, \[ U = U + u' \], m/s

\[ \bar{U} \] Ensemble-averaged value of axial component of instantaneous velocity, \[ \bar{U} = \frac{1}{R} \sum_{r=0}^{R} u_r \], m/s

\[ U_m \] (or U) Steady-state ensemble-averaged cross-sectional mean velocity, \[ U_m = Q / A_{cross} \], m/s

\[ U_m(t) \] Time-dependent cross-sectional mean velocity, m/s

\[ \left| U_{m, os, 1} \right| \] Oscillating component of cross sectional mean velocity for the fundamental first wave in the finite Fourier expansion, m/s

\[ \bar{U}_{m, ta} \] Time-averaged component of cross-sectional mean velocity, m/s

\[ \left| U_{os, n} \right| \] Oscillating component of local cross sectional velocity for the fundamental wave in the finite Fourier expansion, m/s

\[ \left| U_{os, 1} \right| \] Oscillating component of local cross sectional velocity for the fundamental first wave in the finite Fourier expansion, m/s

\[ \bar{U}_{ta} \] Time-averaged component of local velocity at any radial position of the probe, m/s

Greek Letters

\[ \Delta \bar{P}(t) \] Instantaneous pressure drop, Pa

\[ \left| \Delta \bar{P}_{os, 1} \right| \] Oscillating component of pressure drop, Pa

\[ \Delta \bar{P}_{ta} \] Time-averaged component of pressure drop, Pa

\[ \nu \] kinematic viscosity, m^2/s

\[ \rho \] Fluid density, kg/m^3

\[ \lambda_u(t) \] Instantaneous friction factor,

\[ \lambda_u(t) = \frac{8 \bar{\tau}_w(t)}{\rho \bar{U}_m^2(t)} \]

\[ \lambda_{u, ta} \] Time averaged friction factor,

\[ \lambda_{u, ta} = \frac{8}{\rho \bar{U}_{m, ta}^2} \int_0^T \bar{\tau}_w(t) \bar{U}_m(t) dt \]

\[ \tau_w(t) \] Instantaneous wall shear stress, Pa

\[ \omega \] Angular frequency of oscillation, \[ \omega = 2 \pi f \] rad/s

\[ \omega' \] Dimensionless frequency of oscillation, \[ \omega' = R \omega / \nu \]

\[ \sqrt{\omega'} \] Womersley number, \[ \sqrt{\omega'} = R \sqrt{\omega / \nu} \]

Other symbols

\[ \angle \] phase lag
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